CHAPTER 11

Waves

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. A travelling wave has the frequency ν and the particle displacement amplitude A. For the wave the particle velocity amplitude is ---------- and the particle acceleration amplitude is ---------- . *(1983 - 2 Marks)*
2. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is ..... metres. *(1984- 2 Marks)*
3. Two simple harmonic motions are represented by the equations

$$y_1 = 10\sin(3\pi t + \pi/4) \text{ and } y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

Their amplitudes are in the ratio of .... *(1986 - 2 Marks)*
4. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of $0.0075\ m^3$. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become ....Hz. *(1987 - 2 Marks)*
5. The amplitude of a wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t = 0$ and by $y = \frac{1}{[1+(x-1)^2]}$ at $t = 2$ seconds, where x and y are in metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is .... m/s. *(1990 - 2 Marks)*
6. A cylinder resonance tube open at both ends has fundamental frequency F in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is .... *(1992 - 1 Mark)*
7. A bus is moving towards a huge wall with a velocity of $5\ ms^{-1}$. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be..... Hz (Speed of sound in air = $342\ ms^{-1}$) *(1994 - 2 Marks)*

B True/False

1. A man stands on the ground at a fixed distance from a siren which emits sound of fixed amplitude. The man hears the sound to be louder on a clear night than on a clear day. *(1980)*
2. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. *(1984- 2 Marks)*
3. A source of sound with frequency 256 Hz is moving with a velocity V towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall he will hear beats *(1985 - 3 Marks)*

C MCQs with One Correct Answer

1. A cylindrical tube open at both ends, has a fundamental frequency 'f' in air. The tube is dipped vertically in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column in now *(1981- 2 Marks)*

   (a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) f (d) $2f$
2. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is *(1988 - 1 Mark)*

   (a) $a\sin(kx + \omega t)$ (b) $-a\cos(kx - \omega t)$
   (c) $-a\cos(kx + \omega t)$ (d) $-a\sin(kx - \omega t)$
3. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is *(1995S)*

   (a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$
   (c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$
4. A wave disturbance in a medium is described by $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$ where x and y are in metre and t is in second *(1995S)*

(a) A node occurs at x = 0.15 m
(b) An antinode occurs at x = 0.3 m
(c) The speed wave is 5 ms^{-1}
(d) The wave length is 0.3 m

5. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be *(1996 - 2 Marks)*
(a) 1.22v (b) 0.61v (c) 1.50v (d) 0.75v

6. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is *(1996 - 2 Marks)*
(a) 200 Hz (b) 300 Hz (c) 240 Hz (d) 480 Hz

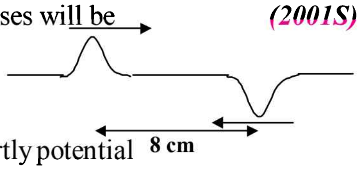
7. A travelling wave in a stretched string is described by the equation $y = A \sin(kx - \omega t)$ The maximum particle velocity is *(1997 - 1 Mark)*
(a) $A\omega$ (b) ω / k (c) $d\omega / dk$ (d) x / t

8. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1/f_2 is *(2000S)*
(a) 18/19 (b) 1/2 (c) 2 (d) 19/18

9. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental nodes, the one of length L with frequency v_1 and the other with frequency v_2. The raio v_1/v_2 is given by *(2000S)*
(a) 2 (b) 4 (c) 8 (d) 1

10. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by *(2000S)*
(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

11. Two pulses in a stretched string whose centers are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be *(2001S)*



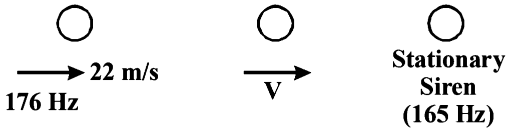
(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential

12. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then *(2001S)*
(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16 E_1$

13. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is
(a) 242/252 (b) 2 *(2002S)*
(c) 5/6 (d) 11/6

14. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is *(2002S)*
(a) 25 kg (b) 5 kg (c) 12.5 kg (d) 1/25 kg

15. A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observes any beats. *(2003S)*



(a) 33m/s (b) 22m/s (c) zero (d) 11m/s

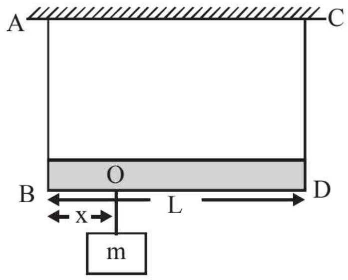
16. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction. *(2003S)*
(a) 0.012m (b) 0.025m (c) 0.05m (d) 0.024m

17. A pipe of length ℓ_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal *(2004S)*
(a) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (b) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$
(c) $\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (d) $\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$

18. In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is *(2005S)*
(a) 51.2 cm/s (b) 102.4 cm/s
(c) 204.8 cm/s (d) 153.6 cm/s

19. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option *(2005S)*
(a) $n = 3,\ f_2 = \frac{3}{4}f_1$ (b) $n = 3,\ f_2 = \frac{5}{4}f_1$
(c) $n = 5,\ f_2 = \frac{3}{4}f_1$ (d) $n = 5,\ f_2 = \frac{5}{4}f_1$

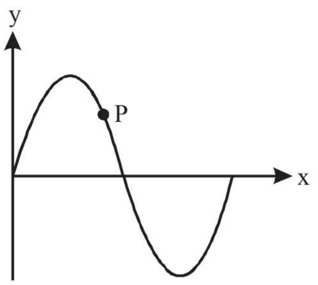
20. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. 'x' is *(2006 - 3M, –1)*



(a) $\frac{L}{5}$
(b) $\frac{4L}{5}$
(c) $\frac{3L}{4}$
(d) $\frac{L}{4}$

21. In the experiment to determine the speed of sound using a resonance column, *(2007)*
(a) prongs of the tuning fork are kept in a vertical plane
(b) prongs of the tuning fork are kept in a horizontal plane
(c) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
(d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

22. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap–shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is – *(2008)*



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$
(b) $-\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$
(c) $\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$
(d) $-\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$

23. A vibrating string of certain length ℓ under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is *(2008)*
(a) 344 (b) 336 (c) 117.3 (d) 109.3

24. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is
(a) 5 grams (b) 10 grams *(2010)*
(c) 20 grams (d) 40 grams

25. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is *(2011)*
(a) 8.50 kHz (b) 8.25 kHz
(c) 7.75 kHz (d) 7.50 kHZ

26. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is *(2012)*
(a) 14.0 cm (b) 15.2 cm (c) 16.4 cm (d) 17.6 cm

D MCQs with One or More than One Correct

1. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4} \sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave *(1982 - 3 Marks)*
(a) travelling with a velocity of 30 m/s in the negative x direction
(b) of wavelength πm
(c) of frequency $30/\pi$ hertz
(d) of amplitude 10^{-4} m traveling along the negative x-direction

2. A transverse wave is described by the equation $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity if *(1984- 2 Marks)*
(a) $\lambda = \pi\frac{y_0}{4}$ (b) $\lambda = \pi\frac{y_0}{2}$
(c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$

3. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is : *(1985 - 2 Marks)*
(a) 31.25 (b) 62.50 (c) 93.75 (d) 125

4. A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends the fundamental frequency that can be excited is (in Hz) *(1986 - 2 Marks)*
(a) 1024 (b) 512 (c) 256 (d) 128

5. The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion are : *(1987 - 2 Marks)*
(a) $\cos kx \sin \omega t$ (b) $k^2x^2 - \omega^2t^2$
(c) $\cos^2(kx + \omega t)$ (d) $\cos(k^2x^2 - \omega^2t^2)$

6. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is *(1988 - 2 Marks)*
(a) 8/3 (b) 3/8 (c) 1/6 (d) 1/3

7. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency : *(1989 - 2 Marks)*
(a) 80 Hz (b) 240 Hz (c) 320 Hz (d) 400 Hz

8. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in meters and t is in seconds. The expression represents : *(1990 - 2 Marks)*
(a) a wave travelling in the positive x-direction with a velocity 1.5 m/s.
(b) a wave traveling in the negative x-direction with a velocity 1.5 m/s.
(c) a wave travelling in the negative x-direction having a wavelength 0.2 m.
(d) a wave travelling in the positive x-direction having a wavelength 0.2 m.

9. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension, *(1991 - 2 Marks)*
(a) T_2 was decreased (b) T_2 was increased
(c) T_1 was decreased (d) T_1 was increased

10. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$

This expression may be consideered to be a result of the superposition of *(1992 - 2 Marks)*
(a) two (b) three (c) four (d) five

11. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is C *(1995S)*
(a) The number of wave striking the surface per second is $f\frac{(c+v)}{c}$
(b) The wavelength of reflected wave is $\frac{c(c-v)}{f(c+v)}$
(c) The frequency of the reflected wave is $f\frac{(c+v)}{(c-v)}$
(d) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vf}{c-v}$

12. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is *(1998 - 2 Marks)*
(a) 0.05 s (b) 0.10 s (c) 0.20 s (d) 0.40 s

13. The (x, y) co-ordinates of the corners of a square plate are $(0, 0)$, $(L, 0)$, (L, L) and $(0, L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is (are) (a = positive constant) *(1998 - 2 Marks)*
(a) $a\cos(\pi x/2L)\cos(\pi y/2L)$ (b) $a\sin(\pi x/L)\sin(\pi y/L)$
(c) $a\sin(\pi x/L)\sin(2\pi y/L)$ (d) $a\cos(2\pi x/L)\sin(\pi y/L)$

14. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$m and $v = 10$ m s^{-1}, then λ and f are given by *(1998 - 2 Marks)*
(a) $\lambda = 2\pi \times 10^{-2}$ m (b) $\lambda = 10^{-3}$ m
(c) $f = 10^3\,Hz/(2\pi)$ (d) $f = 10^4\ Hz$

15. $y(x, t) = 0.8/[4x+5t)^2+5]$ represents a moving pulse, where x and y are in meter and t in second. Then *(1999 - 3 Marks)*
(a) pulse is moving in $+x$ direction
(b) in 2 s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a sysmmetric pulse

16. In a wave motion $y = a\sin(kx - \omega t)$, y can represent *(1999 - 3 Marks)*
(a) electric field (b) magnetic field
(c) displacement (d) pressure

17. Standing waves can be produced *(1999 - 3 Marks)*
(a) on a string clamped at both the ends.
(b) on a string clamped at one end free at the other
(c) when incident wave gets reflected from a wall
(d) when two identical waves with a phase difference of π are moving in the same direction

18. As a wave propagates, *(1999 - 3 Marks)*
(a) the wave intensity remains constant for a plane wave
(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

19. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then, *(2009)*
(a) the intensity of the sound heard at the first resonance was more than that at the second resonance
(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(d) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

20. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe.
When this pulse reaches the other end of the pipe, *(2012)*
(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

21. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01 \text{ m}) \sin[(62.8 \text{ m}^{-1})x] \cos[(628 \text{ s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are) *(JEE Adv. 2013)*
(a) The number of nodes is 5
(b) The length of the string is 0.25 m
(c) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m
(d) The fundamental frequency is 100 Hz

22. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are) *(JEE Adv. 2013)*
(a) If the wind blows from the observer to the source, $f_2 > f_1$
(b) If the wind blows from the source to the observer, $f_2 > f_1$
(c) If the wind blows from observer to the source, $f_2 < f_1$
(d) If the wind blows from the source to the observer, $f_2 < f_1$

23. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

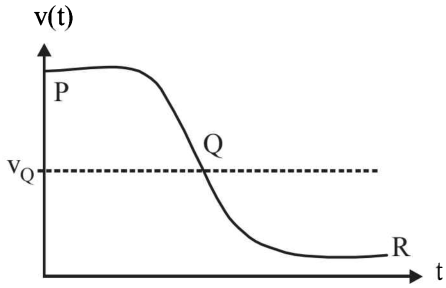
(**Useful information:** $\sqrt{167RT} = 640 J^{1/2} mole^{-1/2}$; $\sqrt{140RT} = 590 J^{1/2} mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.) *(JEE Adv. 2014)*

(a) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$
(b) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$
(c) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$
(d) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

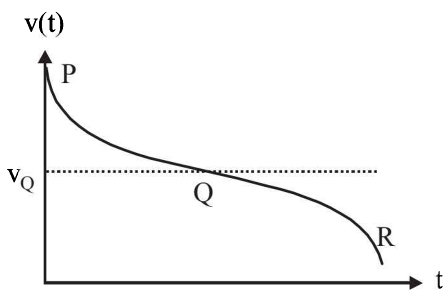
24. One end of a taut string of length 3 m along the x-axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) *(JEE Adv. 2014)*

(a) $y(t) = A \sin\frac{\pi x}{6} \cos\frac{50\pi t}{3}$
(b) $y(t) = A \sin\frac{\pi x}{3} \cos\frac{100\pi t}{3}$
(c) $y(t) = A \sin\frac{5\pi x}{6} \cos\frac{250\pi t}{3}$
(d) $y(t) = A \sin\frac{5\pi x}{2} \cos 250\pi t$

25. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let v(t) represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person? *(JEE Adv. 2016)*
(a) $v_P + v_R = 2 v_Q$
(b) The rate of change in beat frequency is maximum when the car passes through Q
(c) The plot below represents schematically the variation of beat frequency with time

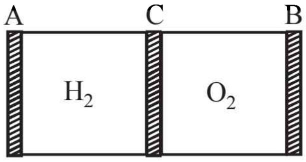


(d) The plot below represents schematically the variation of beat frequency with time



E Subjective Problems

1. *AB* is a cylinder of length 1m fitted with a thin flexible diaphragm *C* at the middle and other thin flexible diaphragms *A* and *B* at the ends. The portions *AC* and *BC* contain hydrogen and oxygen gases respectively. The diaphragms *A* and *B* are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which diaphragm *C* is a node? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). ***(1978)***



2. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C.

   Given : Young modulus of copper $= 1.3 \times 10^{11}$ N/m^2.

   Coefficient of linear expansion of copper $= 1.7 \times 10^{-5}$ °C^{-1}.

   Density of copper $= 9 \times 10^3$ kg/m^3. ***(1979)***

3. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/sec. Estimate the diameter of the tube. ***(1980)***

   One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.

4. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec? ***(1981 - 4 Marks)***

5. A string 25 cm long and having a mass of 2.5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. ***(1982 - 7 Marks)***

6. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope? ***(1984 - 6 Marks)***

7. A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20° C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration.

   Given for steel $Y = 2 \times 10^{11}$ N/m^2

   $\alpha = 1.21 \times 10^{-5}$ per °C ***(1984 - 6 Marks)***

8. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation—

   $y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\,\pi t)$ ***(1985 - 6 Marks)***

   Where *x* and *y* are in cm and *t* in seconds.
   (i) What is the maximum displacement of a point at $x = 5$ cm?
   (ii) Where are the nodes located along the string?
   (iii) What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ sec.?
   (iv) Write down the equations of the component waves whose superposition gives the above wave

9. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. ***(1986 - 8 Marks)***

10. The following equations represent transverse waves :

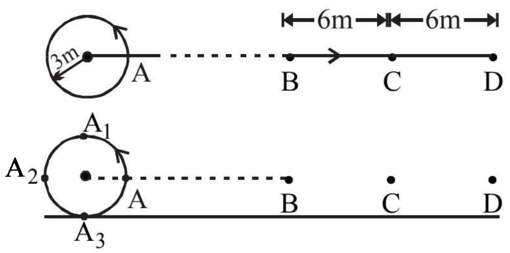
    $z_1 = A\cos(kx - \omega t)$; ***(1987 - 7 Marks)***

    $z_2 = A\cos(kx + \omega t)$; $z_3 = A\cos(ky - \omega t)$

    Identify the combination (s) of the waves which will produce (i) standing wave (s), (ii) a wave travelling in the directon making an angle of 45° degrees with the positive *x* and positive *y* axes. In each case, find the positions at which the resultant intensity is always zero.

11. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train Find ***(1988 - 5 Marks)***
    (i) the Frequency of the whistle as heard by an observer on the hill,
    (ii) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.
    (Velocity of sound in air =1,200 km/hr)

12. A source of sound is moving along a circular orbit of radius 3 metres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line *BD* with an amplitude $BC = CD = 6$ metres. The frequency of oscillation of the detector is $\frac{5}{\pi}$ per second. The source is at the point *A* when the detector is at the point *B*. If the source emits a continous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. ***(1990 - 7 Mark)***



13. The displacement of the medium in a sound wave is given by the equation $y_1 = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. *(1991 - 4 × 2 Marks)*
   (a) What are the wavelength and frequency of incident wave?
   (b) Write the equation for the reflected wave.
   (c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.
   (d) Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave ? What is the direction of propagation of travelling wave?

14. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3$ Hz A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2A^2$. *(1993 - 4 Marks)*
   (i) Find the time interval between successive maxima of the intensity of the signal received by the detector.
   (ii) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

15. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5m length and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle. *(1996 - 3 Marks)*

16. A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist. *(1997 - 5 Marks)*

17. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 m s^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation.
   (a) Find the length L of the air column. *(1998 - 8 Marks)*
   (b) What is the amplitude of pressure variation at the middle of the column?
   (c) What are the maximum and minimum pressures at the open end of the pipe?
   (d) What are the maximum and minimum pressures at the closed end of the pipe?

18. A long wire PQR is made by joining two wires PQ and QR of equal radii PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2 kg. The wire PQR is under a tension of 80 N. A sinusoidal wave-pulse of amplitude 3.5 cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave-pulse. Calculate. *(1999 - 10 Marks)*
   (a) the time taken by the wave-pulse to reach the other end R of the wire, and
   (b) the amplitude of the reflected and transmitted wave-pulses after the incident wave-pulse crosses the joint Q.

19. A 3.6 m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain height in the pipe. Find the height of water level (from the bottom of the pipe) at which resonance occurs. Neglect end correction. Now, the pipe is filled to a height H (≈ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurance of first two resonances. Speed of sound in air is 340 m/s and g = 10 m/s^2. *(2000 - 10 Marks)*

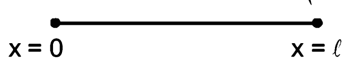
20. A boat is traveling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.
   (a) What will be the frequency detected by a receiver kept inside the river downstream?
   (b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

   (Temperature of the air and water = 20ºC; Density of river water = 10^3 kg/m^3;
   Bulk modulus of the water = 2.088×10^9 Pa; Gas constant $R = 8.31$ J/mol-K;
   Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4) *(2001 - 10 Marks)*

21. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. *(2002 - 5 Marks)*
   (a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.
   (b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

22. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. *(2003 - 2 Marks)*

23. A string tied between $x = 0$ and $x = \ell$ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. *(2003 - 4 Marks)*



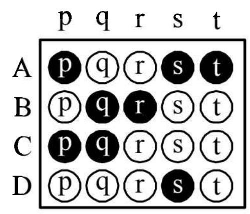
24. A whistling train approaches a junction. An observer standing at junction observes the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train respectively. Find the speed of the train (speed of sound = 300 m/s) ***(2005 - 2 Marks)***

25. A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. ***(2005 - 4 Marks)***

F Match the Following

DIRECTIONS (Q. No. 1-2) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



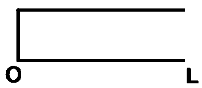
1. Each of the properties of sound listed in the column *A* primarily depends on one of the quantities in column *B*. Write down the matching pairs from the two columns. ***(1980)***

Column A	**Column B**
A. pitch	p. Waveform
B. quality	q. frequency
C. loudness	r. intensity

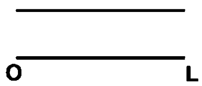
2. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves. ***(2011)***

Column I

(A) Pipe closed at one end

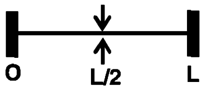


(B) Pipe open at both ends



(C) Stretched wire clamped at both ends

(D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves

(q) Transverse waves

(r) $\lambda_f = L$

(s) $\lambda_f = 2L$

(t) $\lambda_f = 4L$

G Comprehension Based Questions

PASSAGE - 1

Waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling along *x*-axis. (Here *x* is in *m* and *t* is in second)

1. Find the number of times intensity is maximum in time interval of 1 sec. ***(2006 – 5M, –2)***

(a) 4 (b) 6 (c) 8 (d) 10

2. The wave velocity of louder sound is ***(2006 – 5M, –2)***

(a) 100 m/s (b) 192 m/s
(c) 200 m/s (d) 96 m/s

3. The number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 sec is ***(2006 – 5M, –2)***

(a) 100 (b) 46
(c) 192 (d) 96

PASSAGE - 2

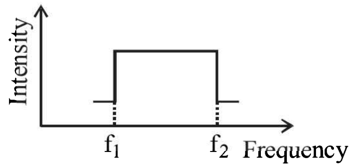
Two trains *A* and *B* moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with *B* ahead of *A*. The engines are at the front ends. The engine of train *A* blows a long whistle.

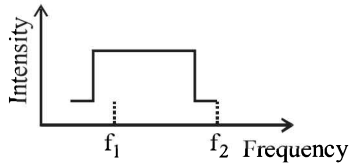
Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.

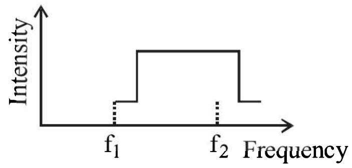
4. The speed of sound of the whistle is ***(2007)***

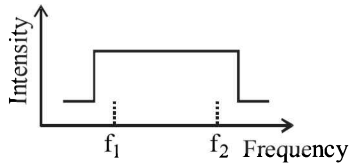
(a) 340 m/s for passengers in *A* and 310 m/s for passengers in *B*
(b) 360 m/s for passengers in *A* and 310 m/s for passengers in *B*
(c) 310 m/s for passengers in *A* and 360 m/s for passengers in *B*
(d) 340 m/s for passengers in both the trains

5. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by *(2007)*

(a)


(b)


(c)


(d)


6. The spread of frequency as observed by the passengers in train B is *(2007)*
(a) 310 Hz (b) 330 Hz
(c) 350 Hz (d) 290 Hz

I Integer Value Correct Type

1. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string. *(2009)*
2. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. *(2010)*
3. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is *(2010)*
4. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}, \frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is **[2002]**
(a) 20 (b) 80 (c) 40 (d) 120.
2. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is **[2002]**
(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1.
3. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is **[2002]**
(a) 286 cps (b) 292 cps (c) 294 cps (d) 288 cps.
4. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is **[2002]**
(a) $y = a\sin(\omega t + kx)$ (b) $y = -a\sin(\omega t + kx)$
(c) $y = a\sin(\omega t - kx)$ (d) $y = -a\sin(\omega t - kx)$
5. When temperature increases, the frequency of a tuning fork **[2002]**
(a) increases (b) decreases
(c) remains same
(d) increases or decreases depending on the material
6. The displacement y of a wave travelling in the x-direction is given by
$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$ metres
where x is expressed in metres and t in seconds. The speed of the wave - motion, in ms^{-1}, is **[2003]**
(a) 300 (b) 600 (c) 1200 (d) 200
7. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is **[2003]**
(a) 50 Hz (b) 100 Hz (c) 200Hz (d) 25Hz
8. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[2003]**
(a) 256 + 2 Hz (b) 256 − 2 Hz
(c) 256 − 5 Hz (d) 256 + 5 Hz
9. The displacement y of a particle in a medium can be expressed as,
$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)m$ where t is in second and x in meter. The speed of the wave is **[2004]**
(a) 20 m/s (b) 5 m/s
(c) 2000 m/s (d) 5π m/s

10. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2? [2005]
(a) 202 Hz (b) 200 Hz (c) 204 Hz (d) 196 Hz

11. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency ? [2005]
(a) 0.5% (b) zero (c) 20% (d) 5 %

12. A whistle producing sound waves of frequencies 9500 HZ and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 HZ, the maximum value of v upto which he can hear whistle is [2006]
(a) $15\sqrt{2}\ ms^{-1}$ (b) $\frac{15}{\sqrt{2}}\ ms^{-1}$
(c) $15\ ms^{-1}$ (d) $30\ ms^{-1}$

13. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is [2006]
(a) 105 Hz (b) 1.05 Hz (c) 1050 Hz (d) 10.5 Hz

14. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of [2007]
(a) 100 (b) 1000 (c) 10000 (d) 10

15. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then [2008]
(a) 18>x (b) x >54
(c) 54 >x>36 (d) 36>x> 18

16. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0s, respectively, then α and β in appropriate units are [2008]
(a) $\alpha = 25.00\pi, \beta = \pi$ (b) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$
(c) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (d) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

17. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, ν, $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be : [2009]
(a) 3 (b) 2 (c) 1 (d) 4

18. A motor cycle starts from rest and accelerates along a straight path at $2 m/s^2$. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}) [2009]
(a) 98m (b) 147m (c) 196m (d) 49m

19. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(m) \sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

The tension in the string is [2010]
(a) 4.0 N (b) 12.5 N (c) 0.5 N (d) 6.25 N

20. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}\,xt\right)}$. This represents a: [2011]
(a) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$
(b) standing wave of frequency $\sqrt{b}$
(c) standing wave of frequency $\frac{1}{\sqrt{b}}$
(d) wave moving in $+x$ direction speed $\sqrt{\frac{a}{b}}$

21. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now : [2012]
(a) f (b) $f/2$ (c) $3f/4$ (d) $2f$

22. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3\ kg/m^3$ and $2.2 \times 10^{11}\ N/m^2$ respectively ? [JEE-Main 2013]
(a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

23. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. [JEE Main 2014]
(a) 12 (b) 8 (c) 6 (d) 4

24. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to : [JEE Main 2015]
(a) 18% (b) 24% (c) 6% (d) 12%

25. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is : [JEE Main 2016]
(take $g = 10\ ms^{-2}$)
(a) $2\sqrt{2}s$ (b) $\sqrt{2}\ s$ (c) $2\pi\sqrt{2}\ s$ (d) 2 s

26. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now : [JEE Main 2016]
(a) 2f (b) f
(c) $\frac{f}{2}$ (d) $\frac{3f}{4}$

11 Waves

Section-A : JEE Advanced/ IIT-JEE

A 1. $A(2\pi\nu), A(2\pi\nu)^2$ 2. 0.125 m 3. 1 : 1 4. 240 Hz 5. 0.5 ms^{-1}
6. f 7. 6 Hz

B 1. F 2. T 3. F

C 1. (c) 2. (c) 3. (a) 4. (c) 5. (a) 6. (a)
7. (a) 8. (d) 9. (d) 10. (b) 11. (b) 12. (c)
13. (b) 14. (a) 15. (b) 16. (b) 17. (b) 18. (c)
19. (d) 20. (a) 21. (a) 22. (a) 23. (a) 24. (b)
25. (a) 26. (b)

D 1. (a,b,c,d) 2. (b) 3. (a, c) 4. (a) 5. (a, c) 6. (c)
7. (a,b,d) 8. (b, c) 9. (b, c) 10. (b) 11. (a, b, c) 12. (b)
13. (b, c) 14. (a, c) 15. (b, c, d) 16. (a,b,c,d) 17. (a, b, c) 18. (a,c,d)
19. (a,d) 20. (b, d) 21. (b, c) 22. (a, b) 23. (d) 24. (a, c, d) 25. (a,b,c)

E 1. 1650 Hz 2. 70 m/s 3. 3.33 cm; 163 Hz 4. 8 5. 27.04 N

6. 0.75 m/s 7. 11 Hz 8. **(i)** 3.46 cm **(ii)** 0, 15, 30 **(iii)** zero **(iv)** $2\sin\left(96\pi t+\frac{\pi x}{15}\right)$ and $-2\sin\left(96\pi t-\frac{\pi x}{15}\right)$

9. 1.5 m/s 10. **(i)** z_1 and z_2; $\frac{(2n+1)\pi}{2K}$ where $n = 0, 1, 2, ...$ **(ii)** z_1 and z_3; $\frac{(2n+1)\pi}{K}$

11. **(i)** 599 Hz **(ii)** 0.935 km, 621 Hz 12. 438.7 Hz, 257.3 Hz

13. **(a)** $\frac{2\pi}{a}, \frac{b}{2\pi}$ **(b)** $y = -0.8A\cos(ax-bt)$ **(c)** 1.8 Ab, 0

(d) $y = -1.6A\sin ax \sin bt + 0.2A\cos(ax+bt)$ $\left[n+\frac{(-1)^n}{2}\right]\frac{\pi}{a}$, $-X$ direction 14. **(i)** $\frac{2\pi}{10^3}$ sec. **(ii)** $\frac{\pi}{2}\times10^{-3}$ sec.

15. 403.3 Hz to 484 Hz 16. $\frac{(v+v_m)\times 2v_b f}{v^2-v_b^2}$

17. **(a)** $\frac{15}{16}$ m **(b)** $\frac{\Delta P_0}{\sqrt{2}}$ **(c)** equal to mean pressure **(d)** $P_0+\Delta P_0$, $P_0-\Delta P_0$ 18. **(a)** 0.14s **(b)** 2.0 cm, 1.5 cm

19. $\frac{-dH}{dt} = (1.11\times10^{-2})\sqrt{H}$, 43 sec. 20. **(a)** 1.007×10^5 Hz, **(b)** 1.03×10^5 Hz 21. **(a)** $\frac{400}{189}$ **(b)** $\frac{3}{4}$

22. 336 m/s 23. $\frac{\pi^2 Ta^2}{4\ell}$ 24. 30 m/s 25. $y = 0.1\sin\left[30t\pm\frac{3}{2}x\pm\phi\right]$

F 1. A → q; B → p; C → r 2. A → p, t; B → p, s; C → q, s; D → q, r

G 1. (a) 2. (c) 3. (d) 4. (b) 5. (a) 6. (a)

I 1. 5 2. 7 3. 5 4. 3

Section-B : JEE Main/ AIEEE

1. (b) 2. (c) 3. (b) 4. (b) 5. (b) 6. (a)
7. (a) 8. (c) 9. (b) 10. (d) 11. (c) 12. (c)
13. (a) 14. (a) 15. (b) 16. (a) 17. (b) 18. (a)
19. (d) 20. (a) 21. (a) 22. (b) 23. (c) 24. (d)
25. (a) 26. (b)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Since $y = A \sin(\omega t - kx)$
Displacement amplitude $= A$ (Max displacement)
Particle velocity, $v = \frac{dy}{dt} = A\omega \cos(\omega t - kx)$
$\therefore$ Velocity amplitude $= A\omega = 2\pi\nu A$
Particle acceleration
$$\text{Acc} = \frac{dv}{dt} = -A\omega^2 \sin(\omega t - kx)$$
$\therefore$ Acceleration (Max acc) amplitude $= A\omega^2 = 4\pi^2\nu^2 A$

2. $c = \nu\lambda \quad \therefore \lambda = \frac{c}{\nu} = \frac{330}{660} = 0.5\,\text{m}$
The rarefaction will be at a distance of
$$\frac{\lambda}{4} = \frac{0.5}{4} = 0.125\,\text{m}$$

3. $y_1 = 10 \sin(3\pi t + \pi/4)$ ...(i)
$y_2 = 5 \sin 3\pi t + 5\sqrt{3} \cos 3\pi t$ ...(ii)
$\therefore\ y_2 = 5 \times 2\left[\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right] = 10 \sin(3\pi t + \pi/3)$
The ratio of amplitudes is $10 : 10 = 1 : 1$

4. $$\frac{\nu_1}{\nu_2} = \frac{\frac{1}{2\ell}\sqrt{\frac{50.7 \times 8}{m}}}{\frac{1}{2\ell}\sqrt{\frac{43.2 \times g}{m}}} \Rightarrow \nu_2 = \nu_1\sqrt{\frac{43.2}{50.7}} = 260\sqrt{\frac{43.2}{50.7}} = 240\,\text{Hz.}$$

5. As $y = \frac{1}{(1+x)^2}$
At $t = 0$ and $x = 0$, we get $y = 1$.
Also at $t = 2$ and $x = 1$, again $y = 1$
The wave pulse has travelled a distance of 1m in 2 sec.
$$\therefore\ v = \frac{1}{2} = 0.5\,\text{ms}^{-1}$$

6. In figure (i)
$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$$
and $f = \frac{c}{\lambda} = \frac{c}{2\ell}$
In figure (ii)
$$\Rightarrow \frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$$
and $f' = \frac{c}{\lambda'} = \frac{c}{2\ell} = f$

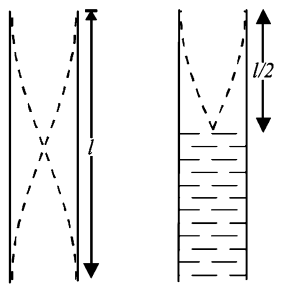
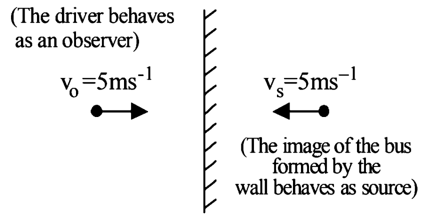


Fig (i) Fig (ii)

7. The first frequency that driver of bus hears is the original frequency of 200 Hz. The second frequency that driver hears is the frequency of sound reflected from the wall. The two frequencies of sound heard by driver is
(a) Original frequency (200 Hz.)
(b) Frequency of sound reflected from the wall (ν')



The frequency of sound reflected from the wall
$$\nu' = \nu\left[\frac{v + v_0}{v - v_s}\right] \Rightarrow \nu' = 200\left[\frac{342 + 5}{342 - 5}\right] \approx 206\,\text{Hz.}$$
$\therefore$ Frequency of beats $= \nu' - \nu = 6\,\text{Hz.}$

B. True/ False

1. The intensity of sound at a given point is the energy per second received by a unit area perpendicular to the direction of propagation.
$$I = \frac{1}{2}\rho V \omega^2 A^2$$
Also intensity varies as distance from the point source as
$$I \propto \frac{1}{r^2}$$
NOTE : None of the parameters are changing in case of a clear night or a clear day.
Therefore the intensity will remain the same.
2. Speed of sound waves in water is greater than in air.
3. **NOTE :** If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source will become the source of reflected sound.
Thus in both the cases, one sound coming directly from the source and the other coming after reflection will have the same apparent frequency (Since velocity of source w.r.t. observer is same in both the cases). Therefore no beats will be heard.

C. MCQs with ONE Correct Answer

1. **(c)** **Case (i)** Here $\frac{\lambda}{2} = \ell \quad \therefore \quad f = \frac{v}{\lambda} = \frac{v}{2\ell}$ ...(i)
Case (ii) Here $\frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell \quad \therefore f = \frac{v}{\lambda'} = \frac{v}{2\ell} = f$
2. **(c)** **NOTE :** Stationary wave is produced when two waves travel in opposite direction.
Now, $y = a\cos(kx - \omega t) - a\cos(kx + \omega t)$
$\therefore$ $y = 2a \sin kx \sin \omega t$ is equation of stationary wave which gives a node at $x = 0$.
3. **(a)** *In air* : $T = mg = \rho V g$
$$\therefore\ f = \frac{1}{2\ell}\sqrt{\frac{\rho V g}{m}} \quad ...(i)$$
In water : $T = mg -$ upthrust
$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

$$\therefore \quad f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho - \rho_\omega)}{m}} = \frac{1}{2\ell}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho-\rho_\omega)}{2\rho}}$$

$$\frac{f'}{f} = \sqrt{\frac{2\rho - \rho_\omega}{2\rho}} \quad f' = f\left(\frac{2\rho-\rho_\omega}{2\rho}\right)^{1/2}$$

$$= 300\left[\frac{2\rho - 1}{2\rho}\right]^{1/2} \text{Hz}$$

4. **(c)** Comparing it with
$y(x, t) = A\cos(\omega t + \pi/2)\cos kx$
If $kx = \pi/2$, a node occurs;
$\therefore \quad 10\pi x = \pi/2 \Rightarrow x = 0.05$ m
If $kx = \pi$, an antinode occurs
$\Rightarrow \quad 10\pi x = \pi \Rightarrow x = 0.1$ m
Also speed of wave

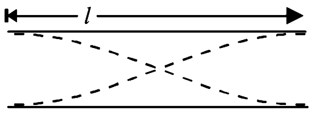
$$= \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ m/s and } \lambda = 2\pi/k = 2\pi/10\pi = 0.2 \text{ m}$$

5. **(a)** Velocity of sound by a stretched string $v = \sqrt{\frac{T}{m}}$

$$\frac{v}{v'} = \sqrt{\frac{T}{T'}} \quad v' = v\sqrt{\frac{T'}{T}} = v\sqrt{\frac{1.5x}{x}} = 1.22\,v$$

6. **(a)** **For both end open**

$$\frac{2\lambda_1}{4} = \ell \Rightarrow \lambda_1 = 2\ell$$



$$v_1 = \frac{c}{\lambda_1} = \frac{c}{2\ell} \ldots \text{(i)}$$

For one end closed

For third harmonic $\frac{3\lambda_2}{4} = \ell \Rightarrow \lambda_2 = \frac{4\ell}{3}$

$$v_2 = \frac{c}{\lambda_2} = \frac{3c}{4\ell} \quad \ldots \text{(ii)}$$

Given $v_2 - v_1 = 100$
From (i) and (ii)

$$\frac{v_2}{v_1} = \frac{3/4}{1/2} = \frac{3}{2}$$

On solving, we get $v_1 = 200$ Hz.

7. **(a)** $v = \frac{dy}{dt} = -A\omega\cos(kx - \omega t) \quad \therefore \quad v_{max} = A\omega$

8. **(d)** $n_1 = n_0\frac{340}{340-34} = \frac{10}{9}n_0;$

$$n_2 = n_0\frac{340}{340-17} = \frac{20}{19}n_0; \quad \frac{n_1}{n_2} = \frac{10}{9}\times\frac{19}{20} = \frac{19}{18}$$

9. **(d)** $n_1 = \frac{1}{2\ell}\sqrt{\left(\frac{T}{4\pi r^2\rho}\right)}$ and $n_2 = \frac{1}{4\ell}\sqrt{\left(\frac{T}{\pi r^2\rho}\right)}$

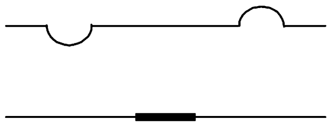
$$n = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}} \quad \left[\text{where } \frac{\lambda}{2} = \text{length of string}\right]$$

$$\therefore \frac{n_1}{n_2} = 2\times\frac{1}{2} = 1 \quad \left[\because m = \frac{\text{mass}}{\text{length}} = \frac{\rho\times A\times\text{length}}{\text{length}} = \rho A\right]$$

10. **(b)** **KEY CONCEPT :** $C_{rms} = \sqrt{\left(\frac{\gamma RT}{M}\right)}$ Here $C_{rms} \propto \sqrt{\frac{1}{m}};$

$$\therefore \quad \frac{C_{rms_1}}{C_{rms_2}} = \sqrt{\left(\frac{m_2}{m_1}\right)}$$

11. **(b)**



After two seconds pulses will overlap each other.
NOTE : According to superposition principle the string will not have any distortion and will be straight. Hence there will be no P.E. The total energy will be only kinetic.

12. **(c)** $E \propto A^2\nu^2$ where A = amplitude and ν = frequency.
Also $\omega = 2\pi\nu \Rightarrow \omega \propto \nu$
In case 1 : Amplitude = A and $\nu_1 = \nu$
In case 2 : Amplitude = A and $\nu_2 = 2\nu$

$$\therefore \quad \frac{E_2}{E_1} = \frac{A^2\nu_2^2}{A^2\nu_1^2} = 4 \Rightarrow E_2 = 4E_1$$

13. **(b)** Using the formula $n' = n\left(\frac{v_A + v}{v}\right)$

$$\frac{v_A + v}{v} = \frac{5.5}{5} \text{ and } \frac{V_B + V}{V} = \frac{6}{5} \Rightarrow \frac{v_B}{v_A} = 2$$

14. **(a)** $f_0 = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$
$\Rightarrow \quad M = 25$ kg
NOTE : Using the formula of a vibrating string,

$$f = \frac{p}{2\ell}\sqrt{\frac{T}{\mu}} \text{ where } p = \text{number of loops.}$$

In each case, the wire vibrates, in resonance with the same tuning fork. Frequency of wire remains same while p and T change.

$$\therefore \quad \frac{p_1}{2\ell}\sqrt{\frac{T_1}{\mu}} = \frac{p_2}{2\ell}\sqrt{\frac{T_2}{\mu}} \text{ or } p_1\sqrt{T_1} = p_2\sqrt{T_2}$$

$$\text{or} \quad \sqrt{\frac{T_2}{T_1}} = \frac{p_1}{p_2}$$

$$\sqrt{\frac{M\times g}{9\times g}} = \frac{5}{3} \text{ or } M = \frac{5\times5\times9}{3\times3} \text{ or } M = 25 \text{ kg.}$$

15. **(b)** f_1 = frequency of the police car heard by motorcyclist,
f_2 = frequency of the siren heard by motorcyclist.

$$f_1 = \frac{330 - v}{330 - 22}\times176; \quad f_2 = \frac{330+v}{330}\times165;$$

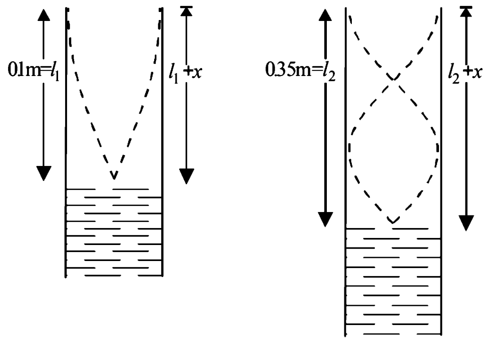
$\because \quad f_1 - f_2 = 0 \Rightarrow v = 22$ m/s

16. **(b)** $\ell_1 + x = \frac{\lambda}{4}$ or, $\lambda = 4(\ell_1 + x)$

$$(\ell_2 + x) = \frac{3\lambda}{4} \text{ or } \lambda = \frac{4}{3}(\ell_2 + x)$$

$$\therefore \quad v_1 = \frac{v}{\lambda_1} = \frac{v}{4(\ell_1 + x)} \quad \therefore \quad v_2 = \frac{v}{\lambda_2} = \frac{3v}{4(\ell_2 + x)}$$

Given $\nu_1 = \nu_2$



or, $\dfrac{v}{4(\ell_1 + x)} = \dfrac{3v}{4(\ell_2 + x)}$ or, $x = 0.025$ m

17. **(b)** Frequency of first overtone in closed pipe,

$$\nu = \frac{3v}{4\ell_1}\sqrt{\frac{P}{\rho_1}} \quad \text{...(i)}$$

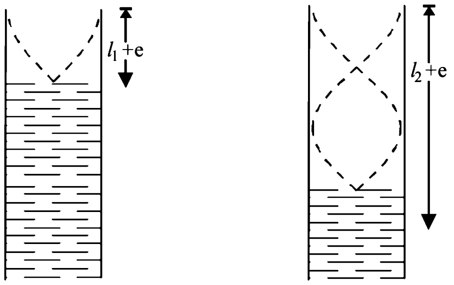
Frequency of first overtone in open pipe,

$$\nu' = \frac{1}{\ell_2}\sqrt{\frac{P}{\rho_2}} \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\Rightarrow \ell_2 = \frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$$

18. **(c)**



For first resonance **For second resonance**

$\ell_1 + e = \dfrac{\lambda}{4}$ $\ell_2 + e = \dfrac{3\lambda}{4}$

But $v = \nu\lambda$

$\therefore v = \nu\dfrac{4}{3}(\ell_2 + e)$ $\Rightarrow \ell_2 + e = \dfrac{3v}{4\nu}$ ...(i)

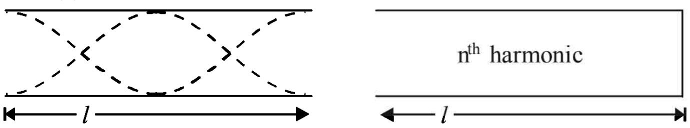
$\therefore v = \nu\, 4(\ell_1 + e)$ $\Rightarrow \ell_1 + e = \dfrac{v}{4\nu}$ ...(ii)

Subtracting (i) and (ii),

$v = 2\nu(\ell_2 - \ell_1)$ $\therefore \Delta v = 2\nu(\Delta\ell_2 + \Delta\ell_1)$

$= 2 \times 512 \times (0.1 + 0.1)$ cm/s $= 204.8$ cm/s

19. **(d)**



$\lambda = \ell$

$\therefore f_1 = \dfrac{v}{\lambda} = \dfrac{v}{\ell}$ ...(i)

$\therefore f_2 = \dfrac{v}{\lambda} = \dfrac{nv}{4\ell}$ ...(ii)

Here n is a odd number. From (i) and (ii)

$f_2 = \dfrac{n}{4}f_1$, For first resonance, $n = 5, f_2 = \dfrac{5}{4}f_1$

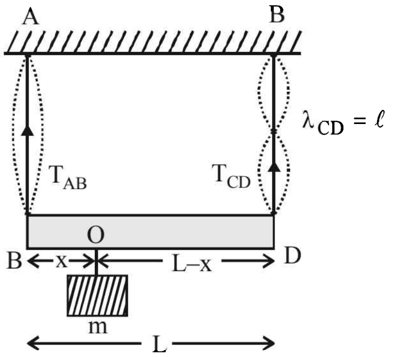
20. **(a)** Frequency of Ist harmonic of $AB = \dfrac{1}{2\ell}\sqrt{\dfrac{T_{AB}}{m}}$

Frequency of 2nd harmonic of $CD = \dfrac{1}{\ell}\sqrt{\dfrac{T_{CD}}{m}}$

Given that the two frequencies are equal.

$$\therefore \frac{1}{2\ell}\sqrt{\frac{T_{AB}}{m}} = \frac{1}{\ell}\sqrt{\frac{T_{CD}}{m}} \Rightarrow \frac{T_{AB}}{4} = T_{CD}$$

$\Rightarrow T_{AB} = 4T_{CD}$ ...(i)



For rotational equilibrium of massless rod, taking torque about point O.

$T_{AB} \times x = T_{CD}(L - x)$ ...(ii)

For translational equilibrium,

$T_{AB} + T_{CD} = mg$ ...(iii)

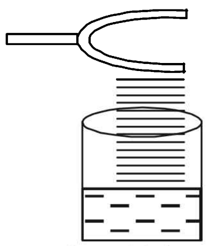
On solving, (i) and (iii), we get

$T_{CD} = \dfrac{mg}{5}$ $\therefore T_{AB} = \dfrac{4mg}{5}$

Substituting these values in (ii), we get

$\dfrac{4mg}{5} \times x = \dfrac{mg}{5}(L - x) \Rightarrow x = \dfrac{L}{5}$

21. **(a)** As shown in the figure, the fringes of the tuning fork are kept in a vertical plane.



22. **(a)** Since the wave is sinusoidal moving in positive x-axis the point will move parallel to y-axis therefore options (c) and (d) are ruled out. As the wave moves forward in positive X-direction, the point should move upwards i.e. in the positive Y-direction. Therefore correct option is a.

23. **(a)** The frequency (ν) produced by the air column is given by

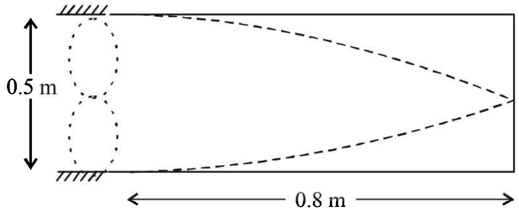
$\lambda \times \nu = v \Rightarrow \nu = \dfrac{v}{\lambda}$

Also, $\dfrac{3\lambda}{4} = \ell = 75\text{cm} = 0.75$ m

$\therefore \lambda = \frac{4 \times 0.75}{3} \Rightarrow v = \frac{340 \times 3}{4 \times 0.75} = 340$ Hz

$\therefore$ The frequency of vibrating string = 340. Since this string produces 4 beats/sec with a tuning fork of frequency n therefore n = 340 + 4 or n = 340 – 4. With increase in tension, the frequency produced by string increases. As the beats/sec decreases therefore n = 340 + 4 = 344 Hz.

24. (b) Frequency of 2nd harmonic of string = Fundamental frequency produced in the pipe



$\therefore 2 \times \left[\frac{1}{2l_1}\sqrt{\frac{T}{\mu}}\right] = \frac{v}{4l_2}$

$\therefore \frac{1}{0.5}\sqrt{\frac{50}{\mu}} = \frac{320}{4 \times 0.8}$

$\therefore \mu = 0.02$ kg m^{-1}

The mass of the string = $\mu\, l_1$

$= 0.02 \times 0.5$ kg = 10g

25. (a) $f' = f\left[\frac{v + v_o}{v - v_s}\right]$

Here v = 320 m/s (given)

$v_o = v_s = 36 \times \frac{5}{18} = 10$ m/s

$f' = 8\left[\frac{320+10}{320-10}\right] = 8 \times \frac{33}{31} \approx 8.5$ kHz

26. (b) Considering the end correction [$e = 0.3\,D$], we get

$L + e = \frac{\lambda}{4} \quad \Rightarrow L = \frac{\lambda}{4} - e$

$= \frac{336 \times 100}{512 \times 4} - 0.3 \times 4 = 15.2$ cm $\left[\because \lambda = \frac{v}{\upsilon}\right]$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(a,b,c,d)** $y = 10^{-4} \sin(60t + 2x)$

Comparing the given equation with the standard wave equation travelling in negative x-direction

$y = a \sin(\omega t + kx)$

we get amplitude $a = 10^{-4}$m

Also, $\omega = 60$ rad/s $\therefore 2\pi f = 60 \Rightarrow f = \frac{30}{\pi}$Hz

Also, $k = 2 \Rightarrow \frac{2\pi}{\lambda} = 2 \Rightarrow \lambda = \pi$ m

We know that $v = f\lambda = \frac{30}{\pi} \times \pi = 30$ m/s

2. **(b)** $y = y_0 \sin 2\pi\left[ft - \frac{x}{\lambda}\right]$

$\therefore \frac{dy}{dt} = \left[y_0 \cos 2\pi\left(ft - \frac{x}{\lambda}\right)\right] \times 2\pi f$

or, $\left[\frac{dy}{dt}\right]_{max} = y_0 \times 2\pi f$

Given that the maximum particle velocity is equal to four times the wave velocity ($c = f \times \lambda$)

$\therefore y_0 \times 2\pi f = 4(f \times \lambda) \therefore \lambda = \frac{\pi y_0}{2}$

3. **(a,c)** The wavelengths possible in an air column in a pipe which has one closed end is

$\lambda = \frac{4\ell}{(2n+1)}$ So, $c = \nu\lambda$

$300 = 264 \times \frac{4\ell}{2n+1}$

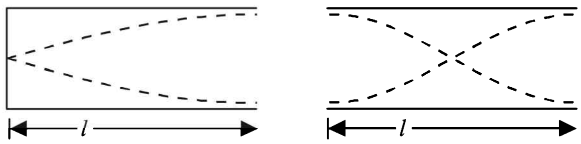
$\nu = 264$ Hz as it is in resonance with a vibrating turning fork of frequency 264 Hz.

$\ell = \frac{330 \times (2n+1)}{264 \times 4}$

For $n = 1$, $\ell = 0.3125$ m = 31.25 cm

For $n = 2$, $\ell = 0.9375$ m = 93.75 cm

4. **(a)**



$\frac{\lambda}{4} = \ell$ (Fundamental mode) $\therefore \lambda = 4\ell$

$\frac{\lambda'}{2} = \ell$ (Fundamental mode) $\therefore \lambda' = 2\ell$

$\therefore \nu = \frac{c}{\lambda} = \frac{c}{4\ell} = 512$ Hz (given)

and $\nu' = \frac{c}{\lambda'} = \frac{c}{2\ell} = 2\left(\frac{c}{4\ell}\right) = 2 \times 512 = 1024$ Hz.

5. **(a,c)** For wave motion, the differential equation is

$\frac{\partial^2 y}{\partial t^2} = \left(\text{constant } \frac{\omega^2}{k^2}\right)\frac{\partial^2 y}{\partial x^2}$

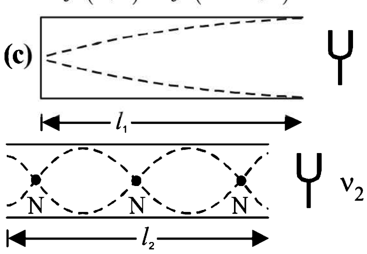
or $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ ....(i)

NOTE : The wave motion is characterized by the two conditions

$f(x, t) = f(x, t + T)$ ....(ii)

$f(x, t) = f(x + \lambda, t)$ ....(iii)

6. **(c)**



$\nu_1 = \frac{v}{4\ell_1}$ for first harmonic

$\nu_2 = \frac{3v}{2\ell_2}$ for third harmonic

$\because \nu_1 = \nu_2 \quad \therefore \frac{v}{4\ell_1} = \frac{3v}{2\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{1}{6}$

7. **(a,b,d)** $\therefore \nu = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}, \ldots = 80, 240, 400 \ldots$

8. **(b,c)** $y = A\sin(10\pi x + 15\pi t + \pi/3)$

The standard equation of a wave travelling in $-X$ direction

is $y = A\sin\left[\frac{2\pi}{\lambda}(vt + x) + (\phi)\right]$

$\Rightarrow y = A\sin\left[\frac{2\pi v}{\lambda}t + \frac{2\pi}{\lambda}x + \phi\right]$

Comparing it with the given equation, we find

$\frac{2\pi v}{\lambda} = 15\pi$ and $\frac{2\pi}{\lambda} = 10\pi$

$\Rightarrow \lambda = \frac{1}{5} = 0.2$ m and $v = \frac{15\pi}{2\pi} \times \frac{1}{5} = 1.5$ m/s

9. **(b,c)** As, $f = \frac{1}{2\pi}\sqrt{\frac{T}{\mu}}$ $\quad\therefore\quad f \propto \sqrt{T}$

Given that $T_1 > T_2$ $\quad\therefore\quad f_1 > f_2$

Initially beat frequency $(f_1 - f_2) = 6$.

The beat frequency remains unchanged which is possible when f_2 increases and f_1 decreases. Thus T_2 increases and T_1 decreases.

10. **(b)** $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t) = 2\left(2\cos^2\frac{t}{2}\sin 1000t\right)$

$= 2[\cos t + 1]\sin 1000t$

$= 2\cos t \sin 1000t + 2\sin 1000t$

$= \sin 1000t + \sin 999t + 2\sin 1000t$

11. **(a, b, c)** Frequency of reflected wave is $f'' = f\left(\frac{c+v}{c-v}\right)$

$\Rightarrow$ Beat freq. $= f'' - f = \frac{2v}{c-1}$.

Wavelength of reflected wave $= \frac{c}{f''} = \frac{c(c-v)}{f(c+v)}$

12. **(b)** **KEY CONCEPT :** The time required for constructive interference equal to the time period of a wave pulse.

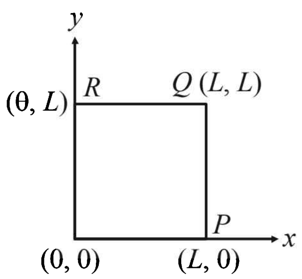
For a string frequency $f = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$

$\therefore$ Time period, $T = 2\ell\sqrt{\frac{m}{F}}$

$F = 1.6$ N, $m = \frac{\text{mass}}{\text{length}} = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2}$

$\therefore \quad T = 2 \times 0.4\sqrt{\frac{2.5 \times 10^{-2}}{1.6}} = 0.1$ sec.

13. **(b,c)** Due to the clamping of the square plate at the edges, its displacements along the x and y axes will individually be zero at the edges. Only the choices (b) and (c) predict these displacements correctly. This is because $\sin 0 = 0$.



Option (a) :

$u(x, y) = 0$ at $x = L, y = L$

$u(x, y) \neq 0$ at $x = 0, y = 0$

Option (b) :

$u(x, y) = 0$ at $x = 0, y = 0 \; [\because \sin 0 = 0]$

$u(x, y) = 0$ at $x = L, y = L \; [\because \sin\pi = 0]$

Option (c) :

$u(x, y) = 0$ at $x = 0, y = 0 \; [\because \sin 0 = 0]$

$u(x, y) = 0$ at $x = L, y = L \; [\because \sin\pi = 0, \sin 2\pi = 0]$

Option (d) :

$u(x, y) = 0$ at $y = 0, y = L \; [\because \sin 0 = 0, \sin\pi = 0]$

$u(x, y) \neq 0$ at $x = 0, x = L \; [\because \cos 0 = 1, \cos 2\pi = 1]$

14. **(a,c)**

NOTE : For a transverse sinusodial wave travelling on a string, the maximum velocity is $a\omega$.

But maximum velocity is $\frac{v}{10} = \frac{10}{10} = 1$ m/s

$\therefore \quad a\omega = 1 \quad \Rightarrow \quad 10^{-3} \times 2\pi v = 1$

$\Rightarrow \quad v = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}$ Hz

$\therefore \quad \lambda = \frac{v}{v} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2}$ m

15. **(b,c,d)** $y = \frac{0.8}{(4x+5t)^2+5} = \frac{0.8}{16\left[x + \frac{5}{4}t\right]^2 + 5}$ ... (1)

We know that equation of moving pulse is

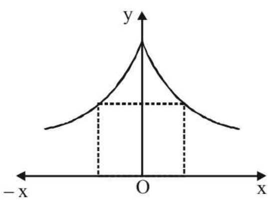
$y = f(x + vt)$ ... (2)

On comparing (1) and (2), we get

$v = \frac{5}{4}\text{ms}^{-1} = \frac{2.5}{2}\text{ms}^{-1}$

So, the wave will travel a distance of 2.5 m in 2 sec.

$\because \; y = \frac{0.8}{(4x+5t)^2+5}$



At

$x = 0, t = 0, y = \frac{0.8}{5} = 0.16$ m

$\therefore$ maximum displacement is 0.16 m

The graph for the given equation is drawn. This is symmetric about y-axis.

16. **(a,b,c,d)** In the wave motion $y = a(kx - \omega t)$, y can represent electric and magnetic fields in electromagnetic waves and displacement and pressure in sound waves.

17. **(a,b,c)** Standing waves are produced by two similar waves superposing while travelling in opposite direction. This can happen in case (a), (b) and (c).

18. **(a,c,d)** For a plane wave, intensity (energy crossing per unit area per unit time) is constant at all points.

But for a spherical wave, intensity at a distance r from a point source of power (P), is given by $I = \frac{P}{4\pi r^2}$

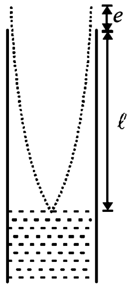
$\Rightarrow\ I \propto \frac{1}{r^2}$

But the **total intensity** of the spherical wave over the spherical surface centered at the source remains constant at all times.

NOTE : For line source $I \propto \frac{1}{r}$

19. (a, d)

At second resonance the length of air column is more as compared to first resonance. Now, longer the length of air column, more is the absorption of energy and lesser is the intensity of sound heard.



As shown in the figure, the length of the air column at the first resonance is somewhat shorter than $\frac{1}{4}$th of the wavelength of the sound in air due to end correction.

$$\ell + e = \frac{\lambda}{4} \qquad \therefore \quad \ell = \frac{\lambda}{4} - e$$

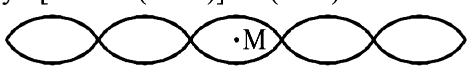
20. (b, d)

When sound pulse is reflected through a rigid boundary (closed end of a pipe), no phase change occurs between the incident and reflected pulse i.e., a high pressure pulse is reflected as a high pressure pulse.

When a sound pulse is reflected from open end of a pipe, a phase change of a radian occurs between the incident and the reflected pulse. A high pressure pulse is reflected as a low pressure pulse.

21. (b, c)

$y = [0.01 \sin(62.8x)] \cos(628\,t)$.



Length of string $= 5 \times \frac{\lambda}{2} = 5 \times \frac{1}{20} = 0.25$ m

$$\left[\because \frac{2\pi}{\lambda} = 62.8\right]$$

The midpoint M is an antinode and has the maximum displacement $= 0.01$ m

The fundamental frequency $= \frac{v}{2l} = \frac{\omega / k}{2l} = 20\text{Hz}$

22. (a,b) Source $S \bullet\!\!\longrightarrow u$ $\quad$ Observer $u \longleftarrow\!\!\bullet O$; f_1 (source), f_2 (observer)

Case1: Wind blows from observer to source

$$f_2 = f_1\left[\frac{(V-w)+u}{(V-w)-u}\right] \qquad \therefore f_2 > f_1$$

Case2 : Wind blows from source to observer

$$f_2 = f_1\left[\frac{(V+w)+u}{(V+w)-u}\right] \qquad \therefore f_2 > f_1$$

(a) and (b) are correct options

23. (d) Here, $v = \frac{v}{4l} = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} \times \frac{1}{4l} \Rightarrow v = v \times 4l$

$\Rightarrow\ v = 336.7$ m/s to 346.5 m/s

For monatomic gas $\gamma = 1.67$

$$v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} = \sqrt{100\gamma RT} \times \sqrt{\frac{10}{M}}$$

$$= \sqrt{167RT} \times \sqrt{\frac{10}{M}} = 640\sqrt{\frac{10}{M}}$$

For Neon $M = 20$ and $\sqrt{\frac{10}{20}} = \frac{7}{10}$

$\therefore\ v = 640 \times \frac{7}{10} = 448\ \text{ms}^{-1}$

$\therefore$ (a) is incorrect

For Argon $M = 36$, $\sqrt{\frac{10}{36}} = \frac{17}{32}$

$\therefore\ v = 640 \times \frac{17}{32} = 340\ \text{ms}^{-1}$

$\therefore$ (d) is the correct option.

For diatomic gas $\gamma = 1.4$

$$v = \sqrt{140RT}\sqrt{\frac{10}{M}} = 590 \times \sqrt{\frac{10}{M}}$$

For Oxygen $\sqrt{\frac{10}{32}} = \frac{9}{16}$

$\therefore\ v = 590 \times \frac{9}{16} = 331.87\ \text{ms}^{-1}$

$\therefore$ (c) is incorrect

For Nitrogen $\sqrt{\frac{10}{28}} = \frac{3}{5}$

$\therefore\ v = 590 \times \frac{3}{5} = 354\ \text{ms}^{-1}$

$\therefore$ (b) is incorrect

24. (a, c, d)

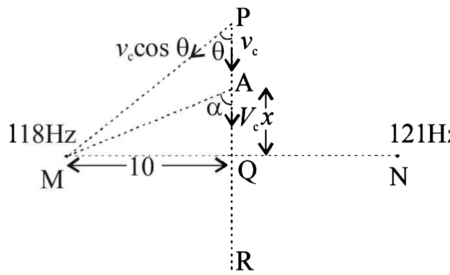
Clearly in the given situation a displacement node is present at $x = 0$ and a displacement antinode is present at $x = 3$ m.

Therefore, $y = 0$ at $x = 0$ and $y = \pm A$ at $x = 3$ m.

The velocity $v = \frac{\omega}{k} = 100\ \text{ms}^{-1}$.

a, c and d are the correct options which satisfy the above conditions.

25. (a,b, c)



$$v_P = 121 - 118 = \left[\frac{v + v\cos\theta}{v}\right]$$

$$v_\theta = 121 - 118 = 3$$

$$v_R = (121-118)\left[\frac{v - v_c \cos\theta}{v}\right]$$

$\therefore\ v_P + v_R = 2v_Q \Rightarrow (A)$ is correct option

In general when the car is passing through A

$$v = 3\left[\frac{v + v_c \cos\alpha}{v}\right] \quad \text{...(i)}$$

$$\therefore \frac{dv}{d\alpha} = -3\left[\frac{v_c \sin\alpha}{v}\right] \quad \left|\frac{dv}{d\alpha}\right| \text{ is max when } \sin\alpha = 1$$

i.e., $\alpha = 90°$ (at Q)

$\Rightarrow$ (b) is correct option.

$$\text{From (i)}\ \frac{dv}{dt} = \frac{3v_c}{v}(-\sin\alpha)\frac{d\alpha}{dt} \quad \text{...(ii)}$$

$$\text{Also } \tan\alpha = \frac{10}{x} \quad \therefore \sec^2\alpha\frac{d\alpha}{dt} = -\frac{10}{x^2}\frac{dx}{dt}$$

$$\therefore \frac{d\alpha}{dt} = \frac{-10v}{x^2\sec^2\alpha} \quad \text{...(iii)}$$

From (ii) & (iii)

$$\frac{dv}{dt} = -\frac{3v_c}{v}\sin\alpha \times \left(\frac{-10v}{x^2\sec^2\alpha}\right) = \frac{30V_c\sin\alpha}{x^2\sec^2\alpha}$$

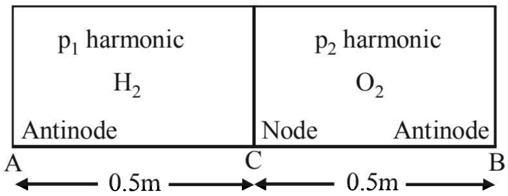
$$\therefore \frac{dv}{dt} = \frac{30v_c\sin\alpha}{(10\cot\alpha)^2\sec^2\alpha} = 0.3v_c\sin^3\alpha \text{ . At } \alpha = 90°$$

$$\frac{dv}{dt} = max$$

$\therefore$ (c) is the correct option

E. Subjective Problems

1. It is given that C acts as a node. This implies that at A and B antinodes are formed. Again it is given that the frequencies are same.



$$\Rightarrow \frac{v_1}{4\ell} \times p_1 = \frac{v_2}{4\ell} \times p_2 \quad \text{or} \quad \frac{p_1}{p_2} = \frac{v_1}{v_2} = \frac{3}{11}$$

or, $11p_1 = 3p_2$

This means that the third harmonic in AC is equal to 11th harmonic in CB.

Now, the fundamental frequency in AC

$$= \frac{v_1}{4\ell} = \frac{1100}{4 \times 0.5} = 550\,\text{Hz}$$

and the fundamental frequency in CB

$$= \frac{v_2}{4\ell} = \frac{300}{4 \times 0.5} = 550\,\text{Hz}$$

$\therefore$ Frequency in $AC = 3 \times 550 = 1650$ Hz

and frequency in $CB = 11 \times 150 = 1650$ Hz

2. **(a)** Using the formula of the coefficient of linear expansion of wire, $\Delta\ell = \ell\alpha\Delta\theta$ we get

$$F = YA\alpha\Delta\theta$$

Speed of transverse wave is given by

$$v = \sqrt{\frac{F}{m}} \quad \left[\text{where } m = \text{mass per unit length} = \frac{Al\rho}{\ell} = A\rho\right]$$

$$= \sqrt{\frac{YA\alpha\Delta\theta}{A\rho}} = \sqrt{\frac{Y\alpha\Delta\theta}{\rho}}$$

$$= \sqrt{\frac{1.3\times10^{11}\times1.7\times10^{-5}\times20}{9\times10^3}} = 70\text{ m/s}$$

3. **Tube open at both ends :**

(a) $v = \dfrac{v}{2(\ell+0.6D)} \quad \therefore 320 = \dfrac{320}{2(0.48+0.6\times D)}$

$0.48 + 0.6D = 0.5 \Rightarrow 0.6D = 0.02$

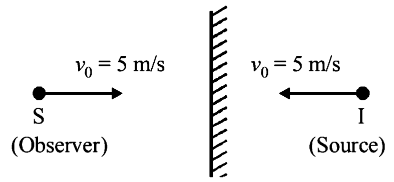
$$\Rightarrow D = \frac{0.02}{60}\times100\text{ cm} = 3.33\text{ cm}$$

Tube closed at one end :

$$v = \frac{v}{4(\ell+0.3D)} = \frac{320}{4(0.48+0.3\times0.033)}$$

≈ 163 Hz

4.



NOTE : If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source in the reflecting surface will become the source of the reflected sound.

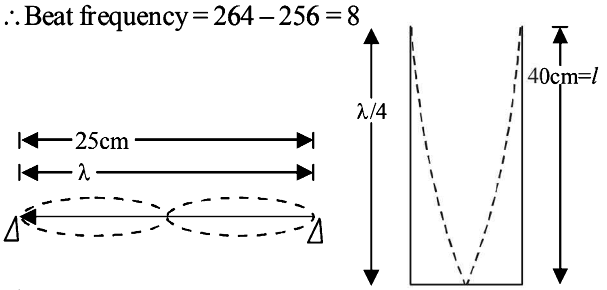
$$v' = v\left[\frac{c - v_0}{c - v_s}\right]$$

v_0, v_s are + ve if they are directed from source to the observer and – ve if they are directed from observer to source.

$$v' = 256\left[\frac{330-(-5)}{330-5}\right] = 264\text{ Hz}$$

$\therefore$ Beat frequency $= 264 - 256 = 8$

5.



First Overtone

$$\text{Mass of string per unit length} = \frac{2.5\times10^{-3}}{0.25} = 0.01\text{ kg/m}$$

$$\therefore \text{Frequency, } v_s = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{1}{0.25}\sqrt{\frac{T}{0.01}} \quad \text{...(i)}$$

Fundamental frequency

$$\therefore \frac{\lambda}{4} = 0.4 \Rightarrow \lambda = 1.6\text{ m}$$

$$\therefore v_T = \frac{c}{\lambda_T} = \frac{320}{1.6} = 200\text{ Hz} \quad \text{...(ii)}$$

Given that 8 beats/second are heard. The beat frequency decreases with the decreasing tension. This means that beat frequency decreases with decreasing ν_s. So beat frequency is given by the expression.

$$\nu = \nu_s - \nu_T$$

$$\therefore \quad 8 = \frac{1}{0.25}\sqrt{\frac{T}{0.01}} - 200 \quad \Rightarrow \quad T = 27.04\,\text{N}$$

6. **KEY CONCEPT :** The velocity of wave on the string is given by the formula

$$v = \sqrt{\frac{T}{m}}$$

where T is the tension and m is the mass per unit length. Since the tension in the string will increase as we move up the string (as the string has mass), therefore the velocity of wave will also increase. (m is the same as the rope is uniform)

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{2\times 9.8}{8\times 9.8}} = \frac{1}{2} \quad \therefore \quad v_2 = 2v_1$$

Since frequency remains the same

$$\therefore \quad \lambda_2 = 2\lambda_1 = 2\times 0.06 = 0.12\,\text{m}$$

7. **KEY CONCEPT :** Using the formula of the coefficient of linear expansion,

$$\Delta\ell = \ell\alpha \times \Delta\theta$$

Also, $$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{\Delta\ell/\ell} = \frac{T/A}{\alpha A\theta} \quad \therefore T = YA\,\alpha\,\Delta\theta$$

The frequency of the fundamental mode of vibration.

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{YA\alpha\,\Delta\theta}{m}}$$

$$= \frac{1}{2\times 1}\sqrt{\frac{2\times 10^{11}\times 10^{-6}\times 1.21\times 10^{-5}\times 20}{0.1}} = 11\,\text{Hz}$$

8. (i) Here amplitude, $A = 4\sin\left(\frac{\pi x}{15}\right)$

At $x = 5$ m

$$A = 4\sin\left(\frac{\pi\times 5}{15}\right) = 4\times 0.866 = 3.46\,\text{cm}$$

(ii) Nodes are the position where $A = 0$

$$\therefore \quad \sin\left(\frac{\pi x}{15}\right) = 0 = \sin n\pi \quad \therefore \quad x = 15\,n$$

where $n = 0, 1, 2$ $\quad x = 15$ cm, 30 cm, 60 cm, ...

(iii) $$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

$$v = \frac{dy}{dt} = 4\sin\left(\frac{\pi x}{15}\right)[-96\pi\sin(96\,\pi t)]$$

At $x = 7.5$ cm, $t = 0.25$ cm

$$v = 4\sin\left(\frac{\pi\times 7.5}{15}\right)[-96\pi\sin(96\,\pi\times 0.25)]$$

$$= 4\sin\left(\frac{\pi}{2}\right)[-96\,\pi\sin(24\pi)] = 0$$

(iv) $$y = 4\sin\left(\frac{\pi x}{15}\right)\cos[96\,\pi t]$$

$$= 2\left[2\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)\right]$$

$$= 2\left[\sin\left(96\,\pi t + \frac{\pi x}{15}\right) - \sin\left(96\,\pi t - \frac{\pi x}{15}\right)\right]$$

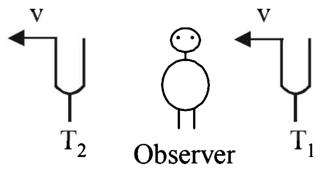
$$= 2\sin\left(96\,\pi t + \frac{\pi x}{15}\right) - 2\sin\left(96\,\pi t - \frac{\pi x}{15}\right)$$

$$= y_1 + y_2$$

where $y_1 = 2\sin\left(96\,\pi t + \frac{\pi x}{15}\right)$

and $y_2 = -2\sin\left(96\,\pi t - \frac{\pi x}{15}\right)$

9. The apparent frequency from tuning fork T_1 as heard by the observer will be



$$\nu_1 = \frac{c}{c - v}\times\nu \qquad \text{... (i)}$$

where c = velocity of sound
v = velocity of turning fork

The apparent frequency from tuning fork T_2 as heard by the observer will be

$$\nu_2 = \frac{c}{c + v}\times\nu \qquad \text{... (ii)}$$

Given $\nu_1 - \nu_2 = 3$

$$\therefore \quad c\times\nu\left[\frac{1}{c-v} - \frac{1}{c+v}\right] = 3 \quad \text{or,} \quad 3 = \frac{c\times\nu\times 2v}{c^2 - v^2}$$

Since, $v << c$ $\quad \therefore \quad 3 = \frac{c\times\nu\times 2v}{c^2}$

$$\therefore \quad v = \frac{3\times 340\times 340}{340\times 340\times 2} = 1.5\,\text{m/s}$$

10. (i) **KEY CONCEPT :** When two progressive waves having same amplitude and period, but travelling in opposite direction with same velocity superimpose, we get standing waves.

The following two equations qualify the above criteria and hence produce standing wave

$$z_1 = A\cos(kx - \omega t)$$
$$z_2 = A\cos(kx + \omega t)$$

The resultant wave is given by $z = z_1 + z_2$

$$\Rightarrow \quad z = A\cos(kx - \omega t) + A\cos(kx + \omega t)$$
$$= 2A\cos kx\cos\omega t$$

The resultant intensity will be zero when

$$2A\cos kx = 0$$

$$\Rightarrow \quad \cos kx = \cos\frac{(2n+1)}{2}\pi$$

$\Rightarrow \; kx = \frac{2n+1}{2}\pi \quad \Rightarrow x = \frac{(2n+1)\pi}{2k}$

where $n = 0, 1, 2, ...$

(ii) The transverse waves

$z_1 = A\cos(kx - \omega t)$

$z_3 = A\cos(ky - \omega t)$

Combine to produce a wave travelling in the direction making an angle of 45° with the positive x and positive y axes.

The resultant wave is given by $z = z_1 + z_3$

$z = A\cos(kx - \omega t) + A\cos(ky - \omega t)$

$$\Rightarrow \; z = 2A\cos\frac{(x-y)}{2}\cos\left[\frac{k(x+y)-2\omega t}{2}\right]$$

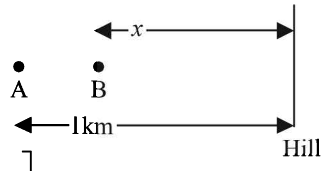
The resultant intensity will be zero when

$$2A\cos\frac{k(x-y)}{2} = 0 \quad \Rightarrow \quad \cos\frac{k(x-y)}{2} = 0$$

$$\Rightarrow \; \frac{k(x-y)}{2} = \frac{2n+1}{2}\pi \quad \Rightarrow \quad (x-y) = \frac{(2n+1)}{k}\pi$$

11. (i) The frequency of the whistle as heard by observer on the hill

$$n' = n\left[\frac{v+v_m}{v+v_m-v_s}\right]$$



$$= 580\left[\frac{1200+40}{1200+40-40}\right] = 599 \text{ Hz}$$

(ii) Let echo from the hill is heard by the driver at B which is at a distance x from the hill.

The time taken by the driver to reach from A to B

$$t_1 = \frac{1-x}{40} \quad ...(i)$$

The time taken by the echo to reach from hill

$t_2 = t_{AH} + t_{HB}$

$$t_2 = \frac{1}{(1200+40)} + \frac{x}{(1200-40)} \quad ...(ii)$$

where t_{AH} = time taken by sound from A to H with velocity $(1200+40)$

t_{HB} = time taken by sound from H to B with velocity $1200-40$

From (i) and (ii)

$$t_1 = t_2 \Rightarrow \frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$$

$\Rightarrow \; x = 0.935$ km

The frequency of echo as heard by the driver can be calculated by considering that the source is the acoustic image.

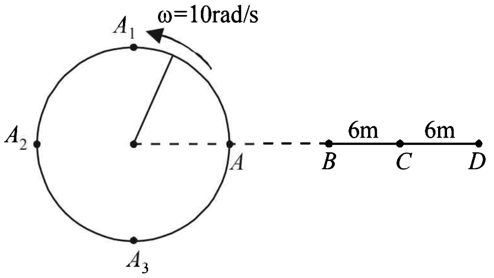
$$n'' = n\left[\frac{(v-v_m)+v_s}{(v-v_m)-v_0}\right]$$

$$= 580\left[\frac{(1200-40)+40}{(1200-40)-40}\right] = 621 \text{ Hz}$$

12. The angular frequency of the detector $= 2\pi\nu$

$$= 2\pi \times \frac{5}{\pi} = 10 \text{ rad/s}$$

The angular frequency of the detector matches with that of the source.



$\Rightarrow$ When the detector is at C moving towards D, the source is at A_1 moving leftwards. It is in this situation that the frequency heard is minimum

$$\nu' = \nu\left[\frac{v-v_0}{v+v_s}\right] = 340 \times \frac{(340-60)}{(340+30)} = 257.3 \text{Hz}$$

Again when the detector is at C moving towards B, the source is at A_3 moving rightward. It is in this situation that the frequency heard is maximum.

$$\nu'' = \nu\left[\frac{v+v_0}{v-v_s}\right] = 340 \times \frac{(340+60)}{(340-30)} = 438.7 \text{Hz}$$

13. **(a) KEY CONCEPT :** Use the equation of a plane progressive wave which is as follows.

$$y = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\nu t\right)$$

The given equation is

$y_1 = A\cos(ax + bt)$

On comparing, we get $\frac{2\pi}{\lambda} = a \;\Rightarrow\; \lambda = \frac{2\pi}{a}$

Also, $2\pi\nu = b$

$$\Rightarrow \; \nu = \frac{b}{2\pi}$$

(b) Since the wave is reflected by an obstacle, it will suffer a phase difference of π. The intensity of the reflected wave is 0.64 times of the incident wave.

Intensity of original wave $I \propto A^2$

Intensity of reflected wave $I' = 0.64\, I$

$\Rightarrow \; I' \propto A'^2 \Rightarrow 0.64\, I \propto A'^2$

$\Rightarrow \; 0.64\, A^2 \propto A'^2 \Rightarrow A' \propto 0.8A$

So the equation of resultant wave becomes

$y_2 = 0.8A\cos(ax - bt + \pi) = -0.8\, A\cos(ax - bt)$

(c) KEY CONCEPT : The resultant wave equation can be found by superposition principle

$y = y_1 + y_2$

$= A\cos(ax + bt) + [-0.8\, A\cos(ax - bt)]$

The particle velocity can be found by differentiating the above equation

$$v = \frac{dy}{dt} = -Ab\sin(ax+bt) - 0.8\, Ab\sin(ax - bt)$$

$= -Ab\,[\sin(ax+bt) + 0.8\sin(ax-bt)]$

$= -Ab\,[\sin ax\cos bt + \cos ax\sin bt$
$\quad + 0.8\sin ax\cos bt - 0.8\cos ax\sin bt]$

$v = -Ab\,[1.8\sin ax\cos bt + 0.2\cos ax\sin bt]$

The maximum velocity will occur when $\sin ax = 1$ and $\cos bt = 1$ under these condition $\cos ax = 0$ and $\sin bt = 0$

$\therefore \quad |v_{max}| = 1.8\, Ab$

Also, $|v_{min}| = 0$

(d) $y = [A\cos(ax+bt)] - [0.8A\cos(ax-bt)]$

$= [0.8A\cos(ax+bt) + 0.2A\cos(ax+bt)] - [0.8A\cos(ax-bt)]$

$= [0.8A\cos(ax+bt) - 0.8A\cos(ax-bt)] + 0.2A\cos(ax+bt)]$

$$= 0.8A\left[-2\sin\left\{\frac{(ax+bt)+(ax-bt)}{2}\right\}\sin\left\{\frac{(ax+bt)-(ax-bt)}{2}\right\}\right] 0.2A\cos(ax+bt)]$$

$\Rightarrow\ y = -1.6A\sin ax\sin bt + 0.2A\cos(ax+bt)$

where $(-1.6A\sin ax\sin bt)$ is the equation of a standing wave and $0.2A\cos(ax+bt)$ is the equation of travelling wave.

The wave is travelling in $-x$ direction.

NOTE : Antinodes of the standing waves are the positions where the amplitude is maximum,

i.e., $\sin ax = 1 = \sin\left[n\pi + (-1)^n\frac{\pi}{2}\right]$

$$\Rightarrow\ x = \left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$$

14. Let the two radio waves be represented by the equations

$y_1 = A\sin 2\pi\nu_1 t$

$y_2 = A\sin 2\pi\nu_2 t$

The equation of resultant wave according to superposition principle

$y = y_1 + y_2 = A\sin 2\pi\nu_1 t + A\sin 2\pi\nu_2 t$

$= A[\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t]$

$$= A\times 2\sin\frac{(2\pi\nu_1 + 2\pi\nu_2)t}{2}\cos\frac{(2\pi\nu_1 + 2\pi\nu_2)t}{2}$$

$= 2A\sin\pi(\nu_1+\nu_2)t\cos\pi(\nu_1-\nu_2)t$

where the amplitude $A' = 2A\cos\pi(\nu_1-\nu_2)t$

Now, intensity $\propto$ (Amplitude)^2

$\Rightarrow\ I \propto A'^2$

$I \propto 4A^2\cos^2\pi(\nu_1-\nu_2)t$

The intensity will be maximum when

$\cos^2\pi(\nu_1-\nu_2)t = 1$

or, $\cos\pi(\nu_1-\nu_2)t = 1$

or, $\pi(\nu_1-\nu_2)t = n\pi$

$$\Rightarrow\ \frac{(\omega_1-\omega_2)}{2}t = n\pi \quad\text{or,}\quad t = \frac{2n\pi}{\omega_1-\omega_2}$$

$\therefore$ Time interval between two maxima

$$\text{or,}\quad \frac{2n\pi}{\omega_1-\omega_2} - \frac{2(n-1)\pi}{\omega_1-\omega_2} \quad\text{or,}\quad \frac{2\pi}{\omega_1-\omega_2} = \frac{2\pi}{10^3}\text{sec}$$

Time interval between two successive maximas is $2\pi\times 10^{-3}$ sec

(ii) For the detector to sense the radio waves, the resultant intensity $\geq 2A^2$

$\therefore$ Resultant amplitude $\geq \sqrt{2}A$

or, $2A\cos\pi(\nu_1-\nu_2)t \geq \sqrt{2}A$

or, $\cos\pi(\nu_1-\nu_2)t \geq \frac{1}{\sqrt{2}}$ or, $\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right] \geq \frac{1}{\sqrt{2}}$

The detector lies idle when the values of $\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right]$ is between 0 and $\frac{1}{\sqrt{2}}$

$\therefore\ \frac{(\omega_1-\omega_2)t}{2}$ is between $\frac{\pi}{2}$ and $\frac{\pi}{4}$

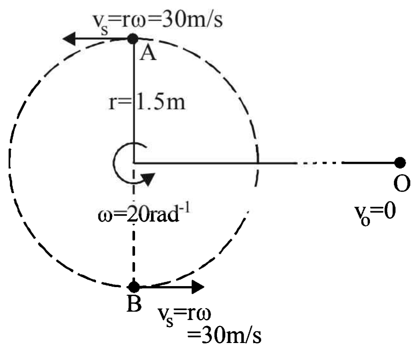
$\therefore\ t_1 = \frac{\pi}{\omega_1-\omega_2}$ and $t_2 = \frac{\pi}{2(\omega_1-\omega_2)}$

$\therefore$ The time gap $= t_1 - t_2$

$$= \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2(\omega_1-\omega_2)}$$

$$= \frac{\pi}{2}\times 10^{-3}\text{ sec}$$

15. The whistle which is emitting sound is being rotated in a circle.



$r = 1.5$ m (given); $\omega = 20$ rads^{-1} (given)

We know that

$v = r\omega = 1.5\times 20 = 30\ \text{ms}^{-1}$

When the source is instantaneously at the position A, then the frequency heard by the observer will be

$$\nu' = \nu\left[\frac{v}{v-v_s}\right] = 440\left[\frac{330}{330-30}\right] = 484\text{ Hz}$$

When the source is instantaneously at the position B, then the frequency heard by the observer will be

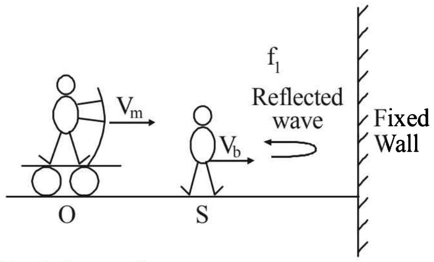
$$\nu'' = \nu\left[\frac{v}{v+v_s}\right] = 440\left[\frac{330}{330+30}\right] = 403.3\text{ Hz}$$

Hence the range of frequencies heard by the observer is 403.3 Hz to 484 Hz.

16. **KEY CONCEPT :** Motorist will listen two sound waves. One directly from the sound source and other reflected from the fixed wall. Let the apparent frequencies of these two waves as received by motorist are f' and f'' respectively.

For Direct Sound : V_m will be positive as it moves towards the source and tries to increase the apparent frequency. V_b will be taken positive as it move away from the observer and hence tries to decrease the apparent frequency value.

$$f' = \frac{v+v_m}{v+v_b}f \quad \text{...(1)}$$



For reflected sound :
For sound waves moving towards stationary observer (i.e. wall), frequency of sound as heard by wall

$$f_1 = \frac{v}{v - v_b} f$$

After reflection of sound waves having frequency f_1 fixed wall acts as a stationary source of frequency f_1 for the moving observer i.e. motorist. As direction of motion of motorist is opposite to direction of sound waves, hence frequency f'' of reflected sound waves as received by the motorist is

$$f'' = \frac{v + v_m}{v} f_1 = \frac{v + v_m}{v - v_b} f \quad \text{...(2)}$$

Hence, beat frequency as heard by the motorist

$$\Delta f = f'' - f' = \left(\frac{v + v_m}{v - v_b} - \frac{v + v_m}{v + v_b}\right) f$$

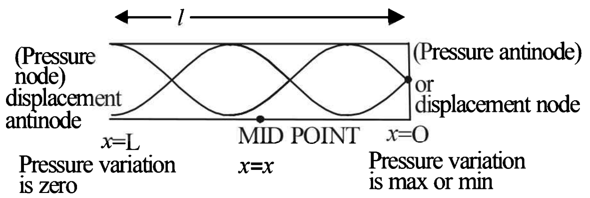
or, $$\Delta f = \frac{2v_b (v + v_m) f}{v^2 - v_b^2}$$

17. (a) For second overtone as shown,

$$\frac{5\lambda}{4} = \ell \quad \therefore \lambda = \frac{4\ell}{5}$$

Also, $\quad v = \nu\lambda$

$$\Rightarrow 330 = 440 \times \frac{4\ell}{5} \Rightarrow \ell = \frac{15}{16} \text{m.}$$



(b) **KEY CONCEPT :** At any position x, the pressure is given by

$$\Delta P = \Delta P_0 \cos kx \cos \omega t$$

Here amplitude $A = \Delta P_0 \cos kx = \Delta P_0 \cos \frac{2\pi}{\lambda} x$

For $x = \frac{15}{2 \times 16} = \frac{15}{32}$ m (mid point)

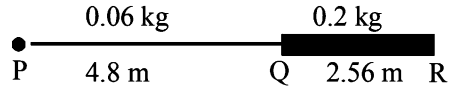
$$\text{Amplitude} = \Delta P_0 \cos\left[\frac{2\pi}{(330/440)} \times \frac{15}{32}\right] = \frac{\Delta P_0}{\sqrt{2}}$$

(c) At open end of pipe, pressure is always same i.e. equal to mean pressure $\because \Delta P = 0, P_{max} = P_{min} = P_0$

(d) At the closed end :
Maximum Pressure $= P_0 + \Delta P_0$
Minimum Pressure $= P_0 - \Delta P_0$

18. (a) Mass per unit length of PQ

$$m_1 = \frac{0.06}{4.8} \text{ kg/m}$$



Mass per unit length of QR, $m_2 = \frac{0.2}{2.56}$ kg/m

Velocity of wave in PQ is

$$v_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{0.06/4.8}} = 80 \text{ ms}^{-1} \ [\because T = 80 \text{ N given}]$$

Velocity of wave in QR is

$$v_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{0.2/2.56}} = 32 \text{ m/s}$$

$\therefore$ Time taken for the wave to reach from P to R

$= t_{PQ} + t_{QR}$

$= \frac{4.8}{80} + \frac{2.56}{32} = 0.14$ s

(b) When the wave which initiates from P reaches Q (a denser medium) then it is partly reflected and partly transmitted.
In this case the amplitude of reflected wave

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_i \quad \text{... (i)}$$

where A_i = amplitude of incident wave.
Also amplitude of transmitted wave is

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i \quad \text{... (ii)}$$

From (i), (ii)
Therefore, $A_t = 2$ cm and $A_r = -1.5$ cm.

19. Speed of sound, $v = 340$ m/s.

Let ℓ_0 be the length of air column corresponding to the fundamental frequency. Then

$$\frac{v}{4\ell_0} = 212.5$$

or $$\ell_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4 \text{ m.}$$

NOTE : In closed pipe only odd harmonics are obtained.

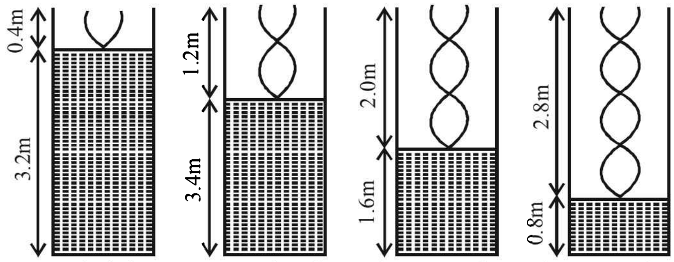
Now, let $\ell_1, \ell_2, \ell_3, \ell_4$, etc. be the lengths corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic etc. Then

$$3\left(\frac{v}{4\ell_1}\right) = 212.5 \Rightarrow \ell_1 = 1.2 \text{ m;}$$

$$5\left(\frac{v}{4\ell_2}\right) = 212.5 \Rightarrow \ell_2 = 2.0 \text{ m}$$

$$7\left(\frac{v}{4\ell_3}\right) = 212.5 \Rightarrow \ell_3 = 2.8 \text{ m;}$$

$$9\left(\frac{v}{4\ell_4}\right) = 212.5 \Rightarrow \ell_4 = 3.6 \text{ m}$$



or heights of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0) m and (3.6 – 2.8) m.

Therefore heights of water level are 3.2 m, 2.4 m, 1.6 m and 0.8 m.

Let A and a be the area of cross-sections of the pipe and hole respectively. Then

$A = \pi(2\times10^{-2})^2 = 1.26\times10^{-3}\,m^{-2}$

and $a = \pi(10^{-3})^2 = 3.14\times10^{-6}\,m^2$

Velocity of efflux, $v = \sqrt{2gH}$

Continuity equation at 1 and 2 gives,

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

Therefore, rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14\times10^{-6}}{1.26\times10^{-3}}\sqrt{2\times10\times H}$$

$$\Rightarrow \frac{-dH}{dt} = (1.11\times10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$$\therefore \frac{dH}{\sqrt{H}} = -1.11\times10^{-2}\,dt$$

$$\Rightarrow \int_{3.2}^{2.4}\frac{dH}{\sqrt{H}} = -(1.11\times10^{-2})\int_0^t dt$$

$$\Rightarrow 2[\sqrt{2.4}-\sqrt{3.2}] = -(1.1\times10^{-2}).t$$

$$\Rightarrow t \approx 43\text{ second}$$

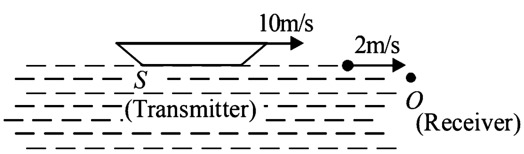
20. **KEY CONCEPT :** The question is based on Doppler's effect where the medium through which the sound is travelling is also in motion.

By Doppler's formula

$$v' = v\left[\frac{c+v_m\pm v_0}{c+v_m\pm v_s}\right] \quad ...(1)$$

NOTE : Sign convention for V_m is as follows :
If medium is moving from s to O then + ve and vice versa.
Similarly v_0 and v_s are positive if these are directed from S to O and vice versa.

(a) **Situation 1.**



Velocity of sound in water $c = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.088\times10^9}{10^3}}$

$c = 1445$ m/s; $v_m = +2$ m/s; $v_0 = 0$; $v_s = 10$ m/s

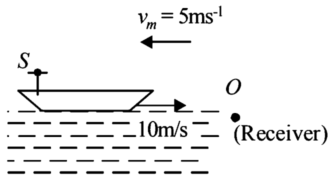
$$\therefore \quad v' = v\left[\frac{1445+2-0}{1445+2-10}\right] = v[1.007]$$

Now, $v = \frac{c}{\lambda} = \frac{1445}{14.45\times10^{-3}} = 10^5$ Hz

$\therefore \quad v' = 1.007\times10^5$ Hz

(b) **Situation 2.**

In air $c = \sqrt{\frac{\gamma RT}{M}} = 344$ m/s



Applying formula (1)

$$v' = v\left[\frac{344-5-0}{344-5-10}\right] = 1.03\times10^5\text{ Hz}$$

21. (a) Second harmonic in pipe A is

$$2(v_0)_A = 2\left[\frac{v}{2\ell}\right] = \frac{1}{\ell}\sqrt{\frac{\gamma_A RT}{M_A}}$$

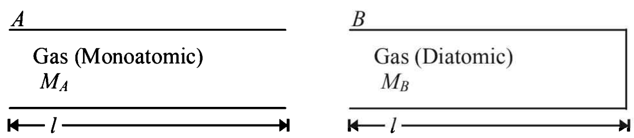
Third harmonic in pipe B is

$$3(v_0)_B = 3\left[\frac{v}{4\ell}\right] = \frac{3}{4\ell}\sqrt{\frac{\gamma_B RT}{M_B}}$$

Given $v_A = v_B$

$$\frac{1}{\ell}\sqrt{\frac{\gamma_A RT}{M_A}} = \frac{3}{4\ell}\sqrt{\frac{\gamma_B RT}{M_B}}$$

or, $\frac{M_A}{M_B} = \frac{\gamma_A}{\gamma_B}\times\left(\frac{4}{3}\right)^2 = \frac{5/3}{7/5}\times\frac{16}{9} = \frac{400}{189}$



Now, $\frac{(v_0)_A}{(v_0)_B} = \sqrt{\frac{\gamma_A}{\gamma_A}\times\frac{M_B}{M_B}} = \frac{3}{4}$

22. **KEY CONCEPT :** In the fundamental mode

$$(\ell+0.6r) = \frac{\lambda}{4} = \frac{v}{4f} \Rightarrow v = 4f(\ell+0.6r) = 336\text{ m/s.}$$

23. Here $\ell = \frac{\lambda}{2}$ or $\lambda = 2\ell$ Since, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\ell} = \frac{\pi}{\ell}$

The amplitude of vibration at a distance x from $x = 0$ is given by $A = a\sin kx$

Mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)A^2\omega^2 = \frac{1}{2}(\mu dx)(a\sin kx)^2(2\pi v)^2$$

$= 2\pi^2\mu v^2 a^2 \sin^2 kx\, dx$

But $v = \nu\lambda$

$\therefore \quad \nu = \frac{v}{\lambda} \Rightarrow \nu^2 = \frac{v^2}{\lambda^2} = \frac{T/\mu}{4\ell^2} \qquad [\because v = \sqrt{T/\mu}]$

$\therefore \quad dE = 2\pi^2\mu \frac{T/\mu}{4\ell^2} a^2 \sin^2\left\{\left(\frac{\pi}{\ell}\right)x\right\}dx$

$\therefore$ Total energy of the string

$$E = \int dE = \int_0^{\ell} 2\pi^2\mu \frac{T/\mu}{4\ell^2} a^2 \sin^2\left(\frac{\pi x}{\ell}\right)dx$$

$$= \frac{\pi^2 T a^2}{4\ell}$$

24. Let the speed of the train be v_T

While the train is approaching

Let ν be the actual frequency of the whistle. Then

$$\nu' = \nu \frac{v_S}{v_S - v_T}$$

where v_S = Speed of sound = 300 m/s (given)

ν' = 2.2 K Hz. = 2200 Hz (given)

$\therefore \quad 2200 = \nu \frac{300}{300 - v_T} \qquad \text{... (i)}$

While the train is receding

$$\nu'' = \nu \frac{v_S}{v_S + v_T}$$

Here, ν' = 1.8 KHz = 1800 Hz (given)

$\therefore \quad 1800 = \nu \frac{300}{300 + v_T} \qquad \text{... (ii)}$

Dividing (i) and (ii)

$\frac{2200}{1800} = \frac{300}{300 - v_T} \times \frac{300 + v_T}{300} \qquad \Rightarrow \quad v_T = 30 \text{ m/s}$

25. **KEY CONCEPT :** The wave form of a transverse harmonic disturbance is

$y = a \sin(\omega t \pm kx \pm \phi)$

Given $v_{max} = a\omega = 3$ m/s ... (i)

$A_{max} = a\omega^2 = 90$ m/s^2 ... (ii)

Velocity of wave $v = 20$ m/s ... (iii)

Dividing (ii) by (i)

$\frac{a\omega^2}{a\omega} = \frac{90}{3} \Rightarrow \omega = 30$ rad/s ... (iv)

Substituting the value of ω in (i), we get

$a = \frac{3}{30} = 0.1\text{m} \qquad \text{... (v)}$

Now, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/\nu} = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2} \qquad \text{... (vi)}$

From (iv), (v) and (vi) the wave form is

$$y = 0.1 \sin\left[30t \pm \frac{3}{2}x \pm \phi\right]$$

F. Match the Following

1. (A) Pitch — q. frequency
(B) quality — p. waveform
(C) loudness — r. intensity

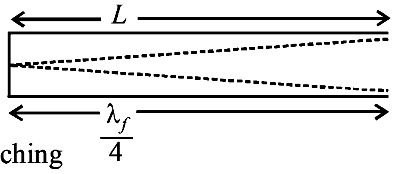
2. **A-p,t; B-p,s; C-q,s; D-q,r**

(A) **Pipe closed at one end**

Waves produced are longitudinal (sound waves)

$\frac{\lambda_f}{4} = L$

$\therefore \lambda_f = 4L$



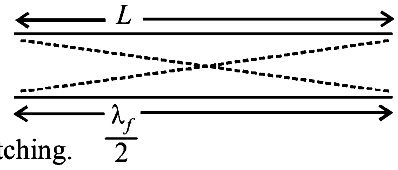
(p, t) are correct matching

(B) **Pipe open at both ends**

waves produced are longitudinal (sound waves)

$\frac{\lambda_f}{2} = L$

$\therefore \lambda_f = 2L$



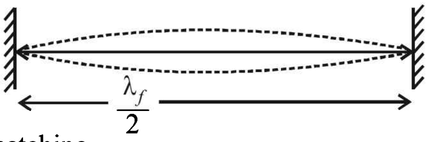
(p, s) are correct matching.

(c) **Stretched wire clamped at both ends**

Waves produced are transverse in nature. (waves on string)

$\frac{\lambda_f}{2} = L$

$\therefore \lambda_f = 2L$

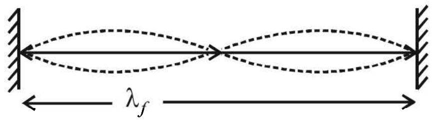


(q, s) are correct matching.

(D) **Stretched wave clamped at both ends & mid point**

Waves produced are transverse in nature (waves on string)

$\lambda_f = L$



(q, r) are correct matching.

G. Comprehension Based Questions

1. **(a)** **2.** **(c)** **3.** **(d)**

The equations are $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ represents two progressive wave travelling in the same direction with slight difference in the frequency. This will give the phenomenon of beats.

Comparing it with the equation

$y = A\cos(kx - \omega t)$, we get

$\omega_1 = 100\pi \Rightarrow 2\pi f_1 = 100\pi \Rightarrow f_1 = 50$ Hz and

$K_1 = 0.5\pi \Rightarrow \frac{2\pi}{\lambda_1} = 0.5\pi \Rightarrow \lambda_1 = 4\text{ m}$

Wave velocity $= \lambda_1 f_1 = 200$ m/s [Alternatively use $v = \frac{\omega}{K}$]

$\omega_2 = 92\pi \Rightarrow 2\pi f_2 = 92\pi \Rightarrow f_2 = 46$ Hz

Therefore beat frequency $= f_1 - f_2 = 4$ Hz and

$K_2 = 0.46\pi \Rightarrow \frac{2\pi}{\lambda_2} = 0.46\pi \Rightarrow \lambda_2 = \frac{200}{46}$

Wave velocity $= \frac{200}{46} \times 46 = 200$ m/s

NOTE : Wave velocity is same because it depends on the medium in which the wave is travelling.

Now, at $x = 0$,

$y_1 + y_2 = (A\cos 10\pi t) + (A\cos 92\pi t) = 0$

$\Rightarrow \cos 100\,\pi t = -\cos 92\,\pi t = \cos(-92\,\pi t)$

$= \cos[(2n+1)\pi - 92\,\pi t \Rightarrow t = \dfrac{2n+1}{192}$

when $t = 0$, $n = -\dfrac{1}{2}$ and when $t = 1$, $n = \dfrac{191}{2} = 95.2$

$\Rightarrow$ net amplitude is zero for n = 96 times (the nearest answer).

4. (b) The speed of sound depends on the frame of reference of the observer.

5. (a) Since all the passengers in train A are moving with a velocity of 20 m/s therefore the distribution of sound intensity of the whistle by the passengers in train A is uniform.

6. (a) $v' = v_1\left[\dfrac{v - v_0}{v - v_s}\right] = 800\left[\dfrac{340-30}{340-20}\right] = 800 \times \dfrac{31}{32}$

$v'' = v_2\left[\dfrac{v - v_0}{v - v_s}\right] = 1120 \times \dfrac{31}{32}$

$\therefore \quad v'' - v' = (1120 - 800) \times \dfrac{31}{32} = 320 \times \dfrac{31}{32} = 310\,\text{Hz}.$

I. Integer Value Correct Type

1. We know that, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{0.5}{10^{-3}/0.2}} = 10\,\text{m/s}$

The wavelength of the wave established

$\lambda = \dfrac{v}{f} = \dfrac{10}{100} = 0.1m = 10\,\text{cm}$

$\therefore$ The distance between two successive nodes

$= \dfrac{\lambda}{2} = \dfrac{10}{2} = 5\text{cm}$

2. Let v be the speed of sound and v_c and f_0 the speed and frequency of car.

The frequency of sound reflected by the car is

$\therefore f_1' = f_0\left[\dfrac{v + v_c}{v - v_c}\right]$

Differentiating the above equation w.r.t. v_c, we get

$$\frac{d f_1'}{dv_c} = f_0\left[\frac{(v - v_c)\dfrac{d}{dv_c}(v + v_c) - (v + v_c)\dfrac{d}{dv_c}(v - v_c)}{(v - v_c)^2}\right]$$

$\therefore \dfrac{df_1'}{dv_c} = f_0\left[\dfrac{2v}{(v - v_c)^2}\right] = f_0\dfrac{2v}{v^2} \quad (\because v_c << v)$

$\therefore \dfrac{df_1'}{f_0} \times 100 = \dfrac{2}{v} \times dv_c$

$\therefore 0.012 = \dfrac{2 \times dv_c}{330} \qquad \therefore dv_c = 0.198\,\text{m/s} \approx$ **7 km/h**

3. Resultant amplitude, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\dfrac{\pi}{2}} = \sqrt{16 + 9 + 0} =$ **5**

4. (3) $y = \sqrt{I_0}\left[\sin O + \sin\dfrac{\pi}{3} + \sin\dfrac{2\pi}{3} + \sin\pi\right]$

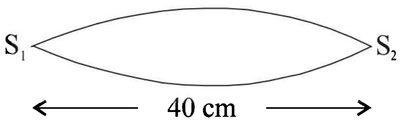
$y = \sqrt{I_0}\left[\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2}\right] = \sqrt{3}\sqrt{I_0}$

$\therefore \quad I_r = y^2 = 3I_0 \quad \Rightarrow \quad n = 3$

Section-B JEE Main/ AIEEE

1. (b) This will happen for fundamental mode of vibration as shown in the figure. S_1 and S_2 are rigid support

Here $\dfrac{\lambda}{2} = 40 \;\therefore \lambda = 80\,\text{cm}$



2. (c) **KEY CONCEPT :** The fundamental frequency for closed organ pipe is given by $\upsilon_c = \dfrac{v}{4\ell}$ and

For open organ pipe is given by $\upsilon_0 = \dfrac{v}{2\ell}$

$\therefore \dfrac{\upsilon_0}{\upsilon_c} = \dfrac{v}{2\ell} \times \dfrac{4\ell}{v} = \dfrac{2}{1}$

3. (b) A tuning fork produces 4 beats/sec with another tuning fork of frequency 288 cps. From this information we can conclude that the frequency of unknown fork is 288 + 4 cps or 288 – 4 cps i.e. 292 cps or 284 cps. When a little wax is placed on the unknown fork, it produces 2 beats/sec. When a little wax is placed on the unknown fork, its frequency decreases and simultaneously the beat frequency decreases confirming that the frequency of the unknown fork is 292 cps.

4. (b) To form a node there should be superposition of this wave with the reflected wave. The reflected wave should travel in opposite direction with a phase change of π. The equation of the reflected wave will be

$y = a\sin(\omega t + kx + \pi)$

$\Rightarrow y = -a\sin(\omega t + kx)$

5. (b) **KEY CONCEPT :** The frequency of a tuning fork is given by the expression

$$f = \frac{m^2 k}{4\sqrt{3}\,\pi\ell^2}\sqrt{\frac{Y}{\rho}}$$

As temperature increases, ℓ increases and therefore f decreases.

6. (a) $y = 10^{-4}\sin\left(600t - 2x + \dfrac{\pi}{3}\right)$

But $y = A\sin(\omega t - kx + \phi)$

On comparing we get $\omega = 600$; k=2

$v = \frac{\omega}{k} = \frac{600}{2} = 300\ ms^{-1}$

7. (a) KEY CONCEPT : For a string vibrating between two rigid support, the fundamental frequency is given by

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\times 1}\sqrt{\frac{10\times 9.8}{9.8\times 10^{-3}}} = 50\,Hz$$

As the string is vibrating in resonance to a.c of frequency n, therefore both the frequencies are same.

8. (c) A tuning fork of frequency 256 Hz makes 5 beats/ second with the vibrating string of a piano. Therefore the frequency of the vibrating string of piano is (256 ± 5) Hz ie either 261Hz or 251 Hz. When the tension in the piano string increases, its frequency will increases. Now since the beat frequency decreases, we can conclude that the frequency of piano string is 251Hz

9. (b) From equation given,

$\omega = 100$ and $k = 20$, $v = \frac{\omega}{k} = \frac{100}{20} = 5 m/s$

10. (d) No. of beats heard when fork 2 is sounded with fork 1 $= \Delta n = 4$

Now we know that if on loading (attaching tape) an unknown fork, the beat frequency increases (from 4 to 6 in this case) then the frequency of the unknown fork 2 is given by,

$n = n_0 - \Delta n = 200 - 4 = 196\,Hz$

11. (c) $n' = n\left[\frac{v+v_0}{v}\right] = n\left[\frac{v+\frac{v}{5}}{v}\right] = n\left[\frac{6}{5}\right]$

$\frac{n'}{n} = \frac{6}{5};\ \frac{n'-n}{n} = \frac{6-5}{5}\times 100 = 20\%$

12. (c) $v' = v\left[\frac{v}{v-v_s}\right] \Rightarrow 10000 = 9500\left[\frac{300}{300-v}\right]$

$\Rightarrow 300 - v = 300\times 0.95 \Rightarrow v = 300 - 285 = 15\ ms^{-1}$

13. (a) Given $\frac{nv}{2\ell} = 315$ and $(n+1)\frac{v}{2\ell} = 420$

$\Rightarrow \frac{n+1}{n} = \frac{420}{315} \Rightarrow n = 3$

Hence $3\times\frac{v}{2\ell} = 315 \Rightarrow \frac{v}{2\ell} = 105$ Hz

Lowest resonant frequency is when $n = 1$

Therefore lowest resonant frequency = 105 Hz.

14. (a) We have, $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$; $L_2 = 10\log\left(\frac{I_2}{I_0}\right)$

$\therefore L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$

or, $\Delta L = 10\log\left(\frac{I_1}{I_0}\times\frac{I_0}{I_2}\right)$ or, $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$

or, $20 = 10\log\left(\frac{I_1}{I_2}\right)$ or, $2 = \log\left(\frac{I_1}{I_2}\right)$

or, $\frac{I_1}{I_2} = 10^2$ or, $I_2 = \frac{I_1}{100}$.

$\Rightarrow$ Intensity decreases by a factor 100.

15. (b) For first resonant length $v = \frac{v}{4\ell_1} = \frac{v}{4\times 18}$ (in winter)

For second resonant length

$v' = \frac{3v'}{4\ell_2} = \frac{3v'}{4x}$ (in summer) $\quad \therefore \frac{v}{4\times 18} = \frac{3v'}{4\times x}$

$\therefore x = 3\times 18\times\frac{v'}{v} \quad \therefore x = 54\times\frac{v'}{v}$ cm

$v' > v$ because velocity of light is greater in summer as compared to winter $(v \propto \sqrt{T})$

$\therefore x > 54$ cm

16 (a) $y(x,t) = 0.005\cos(\alpha x - \beta t)$ (Given)

Comparing it with the standard equation of wave

$y(x,t) = a\cos(kx - \omega t)$ we get

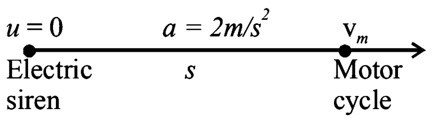
$k = \alpha$ and $\omega = \beta$

$\therefore \frac{2\pi}{\lambda} = \alpha$ and $\frac{2\pi}{T} = \beta$

$\therefore \alpha = \frac{2\pi}{0.08} = 25\pi$ and $\beta = \frac{2\pi}{2} = \pi$

17. (b) Maximum number of beats $= (\nu + 1) - (\nu - 1) = 2$

18. (a)



$v_m^2 - u^2 = 2as \Rightarrow v_m^2 = 2\times 2\times s \quad \therefore v_m = 2\sqrt{s}$

According to Doppler's effect

$0.94v = v\left[\frac{330 - 2\sqrt{s}}{330}\right] \Rightarrow s = 98.01\,m$

19. (d) $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)}\right) - \frac{x}{0.50(m)}\right]$

But $y = a\sin(\omega t - kx)$

$\therefore \omega = \frac{2\pi}{0.04} \Rightarrow \nu = \frac{1}{0.04} = 25\,Hz$

$k = \frac{2\pi}{0.50} \Rightarrow \lambda = 0.5\,m$

$\therefore$ velocity, $v = \nu\lambda = 25\times 0.5$ m/s $= 12.5$ m/s

Velocity on a string is given by

$v = \sqrt{\frac{T}{\mu}} \quad \therefore T = v^2\times\mu = (12.5)^2\times 0.04 = 6.25\,N$

20. **(a)** Given wave equation is $y(x,t)=e^{\left(-ax^2+bt^2+2\sqrt{ab}\,xt\right)}$

$= e^{-[(\sqrt{a}x)^2+(\sqrt{b}t)^2+2\sqrt{a}x.\sqrt{b}t]} = e^{-(\sqrt{a}x+\sqrt{b}t)^2}$

$= e^{-\left(x+\sqrt{\frac{b}{a}}t\right)^2}$

It is a function of type $y=f(x+vt)$

$\Rightarrow$ Speed of wave $=\sqrt{\frac{b}{a}}$

21. **(a)** The fundamental frequency of open tube

$$\nu_0=\frac{v}{2l_0} \quad \ldots(i)$$

That of closed pipe

$$\nu_c=\frac{v}{4l_c} \quad \ldots(ii)$$

According to the problem $l_c=\frac{l_0}{2}$

Thus $\nu_c=\frac{v}{l_0/2}\Rightarrow \nu_c\frac{v}{2l} \quad \ldots(iii)$

From equations (i) and (iii)

$$\nu_0=\nu_c$$

Thus, $\nu_c=f \quad (\because \nu_0=f$ is given)

22. **(b)** Fundamental frequency,

$$f=\frac{v}{2\ell}=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}\left[\because v=\sqrt{\frac{T}{\mu}} \text{ and } \mu=\frac{m}{\ell}\right]$$

Also, $Y=\frac{T\ell}{A\Delta\ell}\Rightarrow\frac{T}{A}=\frac{Y\Delta\ell}{\ell}\Rightarrow f=\frac{1}{2\ell}\sqrt{\frac{\gamma\Delta\ell}{\ell\rho}}$ ....(i)

Putting the value of $\ell,\frac{\Delta\ell}{\ell}$, ρ and γ in eq^n. (i) we get,

$$f=\sqrt{\frac{2}{7}}\times\frac{10^3}{3} \quad \text{or, } f\approx 178.2\text{ Hz}$$

23. **(c)** Length of pipe $=85$ cm $=0.85$m

Frequency of oscillations of air column in closed organ pipe is given by,

$$f=\frac{(2n-1)v}{4L}$$

$$f=\frac{(2n-1)v}{4L}\le 1250$$

$$\Rightarrow \frac{(2n-1)\times 340}{0.85\times 4}\le 1250$$

$$\Rightarrow 2n-1\le 12.5\approx 6$$

24. **(d)** $f_1=f\left[\frac{v}{v-v_s}\right]=f\times\frac{320}{300}Hz$

$$f_2=f\left[\frac{v}{v+v_s}\right]=f\times\frac{320}{340}Hz$$

$$\left(\frac{f_2}{f_1}-1\right)\times 100=\left(\frac{300}{340}-1\right)\times 100\simeq 12\%$$

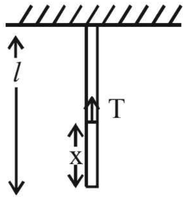
25. **(a)** We know that velocity in string is given by

$$v=\sqrt{\frac{T}{\mu}} \quad \ldots(I)$$

where $\mu=\frac{m}{l}=\frac{\text{mass of string}}{\text{length of string}}$

The tension $T=\frac{m}{\ell}\times x\times g \quad$ ..(II)

From (a) and (b)

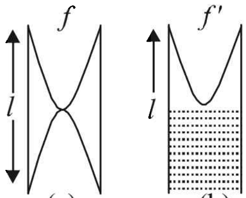


$$\frac{dx}{dt}=\sqrt{gx}$$

$x^{-1/2}dx=\sqrt{g}\,dt \quad \therefore \int_0^{\ell}x^{-1/2}dx-\sqrt{g}\int_0^{\ell}dt$

$2\sqrt{l}=\sqrt{g}\times t \quad \therefore t=2\sqrt{\frac{\ell}{g}}=2\sqrt{\frac{20}{10}}=2\sqrt{2}$

26. **(b)**



(a) (b)

The fundamental frequency in case (a) is $f=\frac{v}{2\ell}$

The fundamental frequency in case (b) is

$$f'=\frac{v}{4(\ell/2)}=\frac{u}{2\ell}=f$$